SOME THEMES AND COMMON TOOLS

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This school concerns topics of classic difficulty: viscous fingering and dendritic growth, membranes and microemulsions, convection and turbulence, fracture and dielectric breakdown. In this opening talk, I wish to argue that we can approach these subjects of classic experimental and theoretical difficulty with the same spirit that has been used in recent years to approach problems associated with phase transitions and critical phenomena. This approach is to carefully choose a microscopic model system that captures the essential physics underlying the phenomena at hand, and then study this model until we understand 'how the model works.' Then we reconsider the phenomena at hand, to see if an understanding of the model leads to an understanding of the phenomena. Sometimes the original model is not enough, and a variant is needed. Fortunately the same underlying physics is often found to be common to both the model and its variants.

1. The Ising Model and Its 'Variants'

We begin, then, with the classic Ising model. Over 1000 papers have been published on this model, but only since 1977 have we known that if one understands the Ising model thoroughly, one understands the essential physics of many materials, since they are simply variants of the Ising model. For example, a large number of systems are related to special cases of the n-vector model, which in turn is a simple Ising model in which the spin variable s has not one component but rather n separate components $s_j$: $s \equiv (s_1, s_2, \ldots, s_n)$.

The Ising model solves the puzzle of how it is that nearest-neighbor interactions of microscopic length scale 1Å 'propagate' their effect cooperatively to give rise to a correlation length $\xi_T$ of macroscopic length scale near the critical point (Fig. 1a). In fact, $\xi_T$ increases without limit as the coupling $K \equiv J/kT$ increases to a critical value $K_c \equiv J/kT_c$,

$$\xi_T \sim A \left( \frac{K - K_c}{K_c} \right)^{-\nu T}.$$

The 'amplitude' $A$ has a numerical value on the order of the lattice constant $a_o$. A snapshot of an Ising system shows that there are fluctuations on all length scales from $a_o (\approx 1\text{Å})$ to $\xi_T$ (which can be from $10^2 - 10^4\text{Å}$ in a typical experiment).

2. Random Site Percolation on a Lattice, and Its Variants

In percolation, one randomly occupies a fraction $p$ of the sites of a d-dimensional lattice (the case $d = 1$ is shown schematically in Fig. 1b). Again, phenomena occurring on the local 1Å scale of a lattice constant are 'amplified' near the percolation threshold $p = p_c$ to a macroscopic length $\xi_p$.

Here $p$ plays the role of the coupling constant $K$ of the Ising model. When $p$ is small, the characteristic length scale is comparable to 1Å. However when $p$
Fig. 1: Schematic illustration of the analogy between (a) the Ising model, which has fluctuations in spin orientation on all length scales from the microscopic scale of the lattice constant $a_0$ up to the macroscopic scale of the thermal correlation length $\xi_T$, (b) percolation, which has fluctuations in characteristic size of clusters on all length scales from $a_0$ up to the diameter of the largest cluster—the pair connectedness length $\xi_p$, and (c) the DLA/DBM problem, whose clusters have fluctuations on all length scales from the microscopic length $d_o = \gamma / L$ ($\gamma$ is the surface tension and $L$ the latent heat) up to the diameter of the cluster $\xi_L$. Also shown, on the right side, is the analogy between the scaling behavior of the three length scales $\xi_T$, $\xi_p$, and $\xi_L$.

approaches $p_c$, there occur phenomena on all scales ranging from $a_0$ to $\xi_p$, where $\xi_p$ increases without limit as $p \to p_c$

$$\xi_p \sim A \left( \frac{p - p_c}{p_c} \right)^{-\nu_p}. \quad (1b)$$

Again, the amplitude $A$ is roughly a lattice constant ($\sim 1\text{Å}$).

Each phenomenon of thermal critical phenomena has a corresponding analog in percolation, so that the percolation problem is sometimes called a geometric or connectivity critical phenomenon. Any connectivity problem can be understood by starting with pure random percolation and then adding interactions, or whatever. Thus, e.g., we understand why the critical exponents describing the divergence to infinity of various geometrical quantities (such as $\xi_p$) are the same regardless of whether the elements interact or are non-interacting. Similarly, the same connectivity exponents are found regardless of whether the elements are constrained to the sites of a lattice or are free to be anywhere in a continuum.

3. The Laplace Equation and Its Variants

Just as variations in the Ising and percolation problems were found to be sufficient to describe a rich range of thermal and geometric critical phenomena, so we have