SIMPLE MODELS FOR COLLOIDAL AGGREGATION, DIELECTRIC BREAKDOWN AND MECHANICAL BREAKDOWN PATTERNS

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ABSTRACT. Some simple models for colloidal aggregation, dielectric breakdown and mechanical breakdown patterns are described. These non-equilibrium growth and aggregation models frequently lead to the formation of complex structures which have a random fractal geometry. They provide a basis for understanding a wide range of phenomena of both practical and scientific importance. In some instances experimental realizations of these simple models have been found. The models discussed in these lectures also provide a sound basis for the development of more elaborate, but more realistic, models which can be applied to a broader range of processes.

In recent years considerable interest has developed in the formation of random patterns under non-equilibrium conditions. A rich phenomenology is found in most pattern formation processes, but in a variety of important cases quite simple models provide a basis for understanding these processes. The purpose of these lectures is to show how simple models can be used to explore the generation of complex (often fractal) patterns by both aggregation and material failure processes.

1. Colloidal Aggregation

The aggregation of small particles to form large structures is important in many systems of both scientific and practical importance. More than 20 years ago simple models for colloidal aggregation were developed by Vold and Sutherland. In recent years a strong resurgence of interest in this approach towards development of a better understanding of non-equilibrium growth and aggregation processes has occurred. This development was stimulated by the discovery of the diffusion-limited aggregation (DLA) model by Witten and Sander which demonstrated that very simple processes could lead to the generation of complex fractal patterns which closely resemble those generated by natural processes. The diffusion-limited aggregation model led to the development of a variety of models for colloidal aggregation including the diffusion-limited cluster-cluster aggregation model and the reaction-limited aggregation model. Despite the fact that these models generate a wide variety of fractal and compact patterns associated with a broad range of physical processes, they are all closely related and can be described in terms of a single general model. In this model we start with a large number \( N_0 \) of particles (single particle clusters) which form a list of clusters. As the simulation proceeds, pairs of clusters are selected from the list and combined irreversibly to form a larger cluster which is returned to the list (which now contains one less cluster). As the simulation proceeds, the clusters get larger and larger and fewer and fewer. The models differ in the way the clusters are selected and the way in which they are combined. In particle-cluster aggregation models such as the Vold-Sutherland (ballistic aggregation), DLA, and Eden models a single particle is always added to...
the largest cluster in the system. The most important characteristic of the way in which the clusters are brought together is the fractal dimension $D_t$ of the trajectories which they follow. The dimension of the trajectory is 2 (diffusion), 1 (ballistic) and 0 for the DLA, Vold-Sutherland and Eden models respectively. Simulations have also been carried out using fractal (Levy flight$^{14}$ and Levy walk$^{15}$) trajectories with dimensions in the range $1 < D_t < 2$. In almost all cases both off-lattice and lattice models have been explored. In most cases they generate clusters with the same fractal dimension (within the accuracy of the simulation). However, for the DLA model lattice anisotropy has an important effect on the cluster shape and fractal dimension$^{16-18}$.

Figure 1 shows clusters generated using two dimensional off-lattice versions of the Eden, Vold-Sutherland and DLA models. The effective fractal dimensions for $d = 2$, 3 and 4 are summarized in Table I.$^{19}$

![Fig. 1: Typical clusters generated using two dimensional off-lattice particle-cluster aggregation models. Except for the dimension of the particle trajectories ($D_t$), these models are very similar. Figure 1a shows a cluster generated using the Eden ($D_t = 0$) model. Figures 1b and 1c show clusters generated using the ballistic aggregation ($D_t = 1$) and diffusion-limited aggregation ($D_t = 2$) models respectively.](image)