THE VISCOPLASTICITY THEORY BASED ON OVERSTRESS APPLIED TO RATCHETTING AND CYCLIC HARDENING

Erhard Krempl and David Yao
Mechanics of Materials Laboratory
Rensselaer Polytechnic Institute
Troy, NY 12180-3590, USA

ABSTRACT

The cyclic neutral version of the theory of viscoplasticity based on over­stress (VBO) is introduced and applied to predict the zero-to-tension ratchetting behavior of a Ti-alloy at room temperature. The experiments show ratchetting to depend on stress rate and this fact together with the accumulated ratchet strains is well represented by VBO. A cyclic hardening version of the theory is shown to correlate the out-of-phase hardening of type 304 stainless steel at room temperature. This is accomplished through the use of two new measures for path length one of which accumu­lates only in nonproportional loading.

NOMENCLATURE

\( \sigma_{ij}, \sigma_{ij} \)  
stress tensor, stress deviator  
\( \varepsilon_{ij}, \varepsilon_{ij} \)  
small strain tensor, small strain deviator  
\( \varepsilon_{ij}, \varepsilon_{ij} \)  
deviatoric equilibrium stress tensor  
\( E \)  
elastic modulus  
\( E_t \)  
tangent modulus of the stress strain curve at the maximum strain of interest; positive, zero or negative  
\( \nu \)  
elastic Poisson's ratio  
\( k[ ] \)  
viscosity function, positive, decreasing; dimension time  
\( \psi[ ] \)  
shape modulus function, positive, decreasing; dimension stress  
\( A \)  
time independent contribution (plastic) contribu­tion to asymptotic stress, dimension stress, see Fig.1  
\( b_1, b_2 \)  
constants, dimension of stress  
\( b_3 \)  
constant, dimensionless  
\( a_1, a_2, a_5, a_5^*, A_0 \)  
constants, dimension of stress  
\( a_4, a_4, a_9, a_{11} \)  
constants, dimensionless  
\( a_{10} \)  
constant, dimension time  
\( \dot{\phi} = \left( \frac{2}{3} \varepsilon_{ij} \varepsilon_{ij} \right)^{1/2} \)  
rate of plastic strain path length  
\( \varepsilon_e = \left( \frac{2}{3} \varepsilon_{ij} \varepsilon_{ij} \right)^{1/2} \)  
asymptotic effective strain  
\( \dot{\varepsilon}_e = \left( \frac{2}{3} \varepsilon_{ij} \varepsilon_{ij} \right)^{1/2} \)  
asymptotic effective strain rate
\[
\sigma^e = \left( \frac{3}{2} \sigma^{in}_{ij} \sigma^{in}_{ij} \right)^{1/2}
\]

asymptotic effective stress

\[
\dot{\sigma}^e = \left( \varepsilon^{in}_{ij} \varepsilon^{in}_{jk} \varepsilon^{ik} \varepsilon^{km} \right)^{1/2}
\]

rate of in-phase path length

\[
\dot{\sigma}^o = \left( \Omega^{ij}_{ij} \right)^{1/2}
\]

rate of out-of-phase path length

\[
\Omega_{ij} = \varepsilon^{in}_{ik} \varepsilon^{in}_{kj} - \varepsilon^{in}_{ik} \varepsilon^{in}_{kj}
\]

tensorial measure of nonproportional loading

INTRODUCTION

It is now generally acknowledged that the computation of deformation behavior of metals and alloys is essential in predicting the life of components subjected to severe conditions of loading and environment. They lead to significant nonlinear, inelastic deformation. Before the advent of the engineering work station in the design office, nonlinear analyses were intractable in engineering and simplified methods of analysis or computations based on a single cycle were performed. The cyclic stress-strain diagram (1) provides a convenient means of performing a monotonic analysis giving rise to a stress state representative for midlife of a component (2-4). The stresses and strains calculated at midlife provide an input to the life analysis under the assumption that they prevail throughout the lifetime of the component. This method is very adequate for cyclic neutral materials but may not lead to the most accurate life prediction if there are cycle dependent property changes manifested in cyclic hardening or softening. Also, such an analysis assumes the existence of a typical cycle throughout the life of a structure which may not be realistic if the actual loading is irregular. The history dependence of deformation may alter the deformation response which may occur for cyclic hardening materials such as annealed stainless steels or copper (5-12)

As the available computing power grows the capacity for economic, nonlinear analyses increases. This power together with the demand for predictability and reliability of performance puts the emphasis on realistic and accurate models of the deformation behavior (the constitutive equation) and on the damage accumulation (life prediction) laws. The area of constitutive laws has been very active recently (13-19), specifically in modeling the phenomena observed with nonproportional loading. The purpose of this paper is to contribute to the discussion by introducing the viscoplasticity theory based on overstress (VBO) in a cyclic neutral and in a cyclic hardening version. A specific application is the modeling of ratchetting for zero-to-tension load controlled loading.

VICOPLASTICITY THEORY BASED ON OVERSTRESS

Recent experiments have shown again that the inelastic deformation behavior of engineering alloys is rate dependent at room temperature. [This fact was already reported by Ludwik (20),] Rate sensitivity, short time creep and relaxation behavior were reported for the following alloys at room temperature: a cyclic neutral Ti-alloy (21), a strongly cyclic hardening, annealed type 304 stainless steel (6,22-24) and a cyclic neutral Al₆-alloy (25,26). Post yield creep of several other alloys was