WATER POLLUTION CONTROL USING FINITE ELEMENT MODEL

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SUMMARY

A finite element model for the water pollution control of a nearly closed water area is described. The principal constraints are to attain the chemical oxygen demand (COD) standards. The finite element method is used for the derivation of linear constraints of the constrained optimization problem. The objective function is employed the total value of removal COD concentration discharging into the water. This model is applied to Tokyo Bay which is a nearly closed water area in Japan.

INTRODUCTION

The combination method of linear programming and finite element methods has proven to be an effective tool for constrained optimization problems. Several investigators have solved the water pollution control or the thermal diffusion control by this method. The finite element method is not only suitable for solving the problems with irregular boundaries but also a useful technique for the derivation of linear constraints of the optimization problem.

In this paper the numerical model is shown to apply to the water pollution control of a nearly closed water area. This model requires the permanent current and substance dispersion patterns. Although tidal residual current is generally used as permanent current, it is difficult to simulated the actual current. The required one is complex. For simplicity, the present study employs the current due to the influence of river discharge. Regarding substance dispersion, the model based on two-dimensional dispersion is used for the calculation of COD concentration. For the numerical simulations of them, different kinds of finite element idealization are employed. Namely, the finite element idealization for the current is smaller than
that for the substance dispersion. Because the smaller the size of finite elements, the better the accuracy, as far as the current is concerned. Moreover, the current is one of the most important factors to determine the dispersion phenomenon.

The model presented in this paper makes it possible to apply to the water pollution control of a lake or a nearly closed water area.

MODEL FORMULATION

In this paper, to get constraints of the linear programming the formulation of the substance dispersion model using the finite element method is applied.

The diffusion-convection equation on COD in an arbitrary domain \( V \) is written as

\[
uc_x + vc_y - D_x c_{xx} - D_y c_{yy} + R c - Q = 0
\]  

(1)

where \( c \) is COD concentration, \( u \) and \( v \) are flow velocities in \( x \)- and \( y \)-directions, respectively, \( D_x \) and \( D_y \) are dispersion coefficients in \( x \)- and \( y \)-directions, \( R \) is substance decaying rate, and \( Q \) is the increasing rate of substance concentration due to a source.

Fig.1 shows a typical domain \( V \) for the analysis. The domain boundary can be classified into two types. They are \( S_1 \) with specified COD concentration and \( S_2 \) with specified flux of concentration.

Boundary conditions on \( S_1 \) and \( S_2 \) are

\[
c = \hat{c} \quad \text{on} \quad S_1 \quad (2)
\]

\[
t = q_x l + q_y m = \hat{t} \quad \text{on} \quad S_2 \quad (3)
\]

where \( \hat{\cdot} \) denotes the specified value on the boundary \( S_1 \) and \( S_2 \), \( q_x \) and \( q_y \) are flux in \( x \)- and \( y \)-directions, \( l \) and \( m \) are directions cosines of the unite outward normal to \( S_2 \) and \( t \) represents flux of concentration.

The Garellkin method is used for the formulation of the finite element method. Using a linear interpolation based on a three-node triangular element, it is obtained that

\[
K_{\alpha\beta}u^x c^y + K_{\alpha\beta} v^y c^x + s_{\alpha\beta} c^\beta + h_{\alpha\beta} c^\beta = \hat{\Omega} \quad (4)
\]