THREE-DIMENSIONAL FINITE ELEMENT ANALYSES FOR A MAXWELL FLUID USING THE PENALTY FUNCTION METHOD
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SUMMARY

The penalty function formulation of the three-dimensional finite element method is applied for analyzing the extrudate swells of a Maxwell fluid. The momentum and the constitutive equations are solved separately until the convergence is achieved, in which the standard Galerkin's method is applied to solve the velocity and the tangential extra-stresses, and the least square finite element method is applied to the normal extra-stresses.

1. INTRODUCTION

In operating polymer processes in the steady-state flows, such as those found in the extrudate molding or the fiber spinning, we often observe a most interesting fact that the shape of a final product can be quite different from that at the die exit[1]. In order to estimate this phenomenon by the numerical simulation, elastic effects should be considered in the equations of governing the flow of viscous fluid. A primary difficulty to analyze such a model is the inability to eliminate stresses from the governing equations. Several approaches are available to overcome this difficulty. Among those, the mixed finite element method which solves velocity, pressure and extra-stresses simultaneously is frequently used[2-5]. Because the method requires a large storage area for the stiffness matrix, the amount of computer time is increased for reducing the matrix. On the other hand, the method which decouples the momentum and continuity equations from the constitutive equation requires a less storage area for the stiffness matrix[6-8]. This method, however, is not only unstable for highly non-linear problems, but also needs a quadratic element for the velocity field. These disadvantages can be improved if we combine the Petrov-Galerkin's method for the extra-stress field with the penalty
function formulation of the standard Galerkin’s method for the velocity field[9-10].

In this study, we describe a compact three-dimensional finite element method for computing the extrudate swells of a Maxwell (upper convected) fluid. The standard Galerkin’s method is applied to solve both the velocity and tangential extra-stresses, and the least square finite element method is applied to the normal extra-stresses. An eight-node isoparametric hexahedral element is used for the approximation of both velocity and extra-stress fields. The pressure replaced by a penalty function is determined with the least square finite element method using the calculated extra-stress tensor and velocity vector[11]. The swells of a planar die are first solved to verify the scheme and check the stability. Both the calculated swells and the numerical stability agree well with each of those obtained by other authors. In the next example, we calculate the swells of the fluid being extruded through a rectangular die. In order to illustrate the swell behavior, the die with several different aspect ratios is considered.

2. GOVERNING EQUATIONS

The governing equations for the slow steady-state flow of an incompressible Maxwell fluid without body forces in rectangular Cartesian coordinates are

Equilibrium: \( \sigma_{ij,j} = 0 \) \hspace{1cm} (1)

Continuity : \( u_{i,i} = \varepsilon_{ii} = 0 \) \hspace{1cm} (2)

 Constitutive relationship:

\[
\sigma_{ij} = \sigma_{ij} + p \delta_{ij} \hspace{1cm} (3)
\]

\[
\varepsilon_{ij} = \frac{1}{2\mu} \sigma_{ij} + \frac{1}{2G} \nabla \cdot \sigma_{ij} \hspace{1cm} (4)
\]

\[
\nabla \cdot \sigma_{ij} = u_k \sigma_{ij,k} - u_j \sigma_{ik}^{\prime} - u_i \sigma_{kj}^{\prime} \hspace{1cm} (5)
\]

\[
\varepsilon_{ij} = \frac{1}{2} ( u_{ij,j} + u_{ij}^{\prime} ) \hspace{1cm} (6)
\]

\[
p = -\frac{1}{3} \sigma_{ii} \hspace{1cm} (7)
\]

Boundary condition:

\( u_j = u_j \) on \( S_u \) \hspace{1cm} (8)

\( \nu_j \sigma_{ij} = \tilde{T}_j \) on \( S_t \) \hspace{1cm} (9)

\( \sigma_{ij} = \tilde{\sigma}_{ij} \) on \( S_\sigma \) \hspace{1cm} (10)