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PHILOSOPHICAL PRAGMATISM IN POINCARE*

At the beginning Poincaré is using the terms 'Intuition' and 'analysis' in order to describe two psychological attitudes involved in the logic of invention: Riemann and Klein represent the attitude of intuition, Hermite and Weierstrass the attitude of analysis. Later on, but usually without explicit indication, these two terms stand likewise for two theories about the nature of mathematical activity: on the one hand you concentrate on investigations into the conditions governing the construction (intuition) of mathematical objects, on the other hand you try to describe (analyze) domains of already existing objects.

In arithmetic and in foundational studies - in this paper I exclude his approach to geometry - Poincaré is almost always an intuitionist. Yet, even though the philosophy of Poincaré remains on the whole an intuitionistic one, he displays analytical features, too. Hence, you find a solution of the problem of predicativity not only by providing limitations to the domains under discussion (these are procedures on the level of construction) but also by providing restrictive clauses with respect to the rules of quantification (these are procedures on the level of description), i.e. prima facie without limiting the domains of quantification. Certainly, here, these two levels correspond and you even arrive at logically equivalent presentations, if you translate the underlying ideas into a formal language. In the first case you end up with an 'exclusive' theory of types, in the second case with a cumulative one.

* This paper, written originally in French, was delivered as a lecture at the University of Aix-en-Provence in October 1985; it derives from an attempt to improve on chapter II of my book: Entre intuition et analyse. Poincaré et le concept de prédicativité, Paris 1985.
However, I want to argue for the claim that the modern idea of a unification of the approach on the level of construction and of the approach on the level of description, these levels being two inseparable aspects of the common ground of actions, can find one of its roots in the philosophical pragmatism of Poincaré. Nevertheless, an attempt of reducing Poincaré's pragmatism to its role as a forerunner of a philosophy conducted in a pragmatist framework, would be a biased approach. Poincaré was first of all an anti-logicist (and anti-formalist), afterwards, as a reaction, he became an intuitionist, and only by retrospection he is found as being placed between 'the methodological fronts'.

In two consecutive papers under the common title 'Les mathématiques et la logique', published in 1905 and 1906, Poincaré argued against the logicist claim of being able to "démontrer toutes les vérités mathématiques ... une fois admis les principes de la logique". For this would mean to give up either the analytical nature of logic or the synthetical nature of mathematics, i.e. to advocate a solution to the problem of defining the relation between logic and mathematics which rests basically on a Leibnitan tradition. Poincaré suspects that during the centennials in honour of Kant's death an equivocation of the term 'logic' is introduced such that 'logic' does not exclusively refer to traditional logic but to a 'new logic' which comprises both synthetic principles of demonstration and the formation of non-logical concepts.

Poincaré sees very clearly here. Predicate logic is not only richer than traditional logic which Kant referred to but in order to deal with reductionism one should also take into account that in the 'new logic' more freedom with respect to certain (set theoretic) existence postulates prevails. For example, is it really an analytical procedure (of the second order) to turn predicates into names and afterwards affirm their existence, i.e. the existence of entities signified? It is obvious that the 'conditional' solution of Russell would not satisfy the constructivist, nor had Poincaré been satisfied.

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1 Cf. Poincaré (1905/1906) and Poincaré (1906).
2 Poincaré (1905/1906) p.817.
3 Poincaré follows Kant in calling propositions analytic when the subject-concept is contained in the predicate-concept.