USE OF THE JACKKNIFE METHOD TO ESTIMATE AUTOCORRELATION FUNCTIONS (OR VARIOGRAMS)

C. F. Chung
Geological Survey of Canada

ABSTRACT

An application of the jackknife method to estimate autocorrelation functions (or variograms) of stationary random functions is discussed. Using the jackknife estimators and the corresponding jackknife variances, the models of the autocorrelation functions are fitted by the weighted least squares method. The method is particularly effective to study robustness of the estimators when the number of data points is small. The technique is applied to simulated data sets with known autocorrelation functions.

INTRODUCTION

To study spatial variation or "regionalized" variables (Matheron, 1962), the theory of stationary random functions has been applied. In application, an important but difficult problem is to postulate the underlying autocorrelation model and to estimate the unknown parameters in the model from the data. This paper does not discuss the problems of how to postulate the model but deals with estimation of its parameters.

Contrary to the importance of the autocorrelation function, only three studies, Agterberg (1970), Huijbregts (1971), and Cressie and Hawkins (1980) have been published on the estimation problems, after their application had been introduced in geosciences and mining engineering by Matheron (1962). However, extensive statistical research have been performed on the estimation of autocorrelation functions in time-series analysis where the observations are made at regular intervals and the

G. Verly et al. (eds.), Geostatistics for Natural Resources Characterization, Part 1, 55–69.
models are relatively simple. The latter two papers have discussed variograms instead of autocorrelation functions. The variogram (Jowett, 1955; Matheron, 1962) is defined as $E(Z(x) - Z(y))^2$ compared to $E((Z(x) - E(Z(x)))(Z(y) - E(Z(y))))$ for autocorrelation function. For a stationary random function,

$$\frac{1}{2} \text{variogram} = \text{variance} - \text{autocorrelation function}$$

Hence, the estimation problems for autocorrelation function and variogram are similar to each other.

Agterberg (1970) has discussed the problem in relation to the irregularly spaced data and data which contain a large scale systematic variation (trend or drift). Huijbregts (1971) has studied how to reconstruct the variogram when the observations are made in intervals (areas in the two dimensional space) rather than at points (i.e. the data are for $\int_A Z(x) \, dx$ where $A$ represents the intervals, instead of for $Z(x)$). Cressie and Hawkins (1980) have discussed several robust methods of estimating variograms and have proposed to use M-estimators after applying the fourth root power transformation.

In this study, the jackknife method is proposed to compute correlograms followed by estimation of the unknown parameters in the autocorrelation models by the weighted least squares technique using jackknife estimators and the corresponding jackknife variances. The jackknife technique is a general method to estimate variances of the resulting estimators and it can be used in conjunction with any estimation procedure.

In order to illustrate the methodology discussed here, two realizations of standard normal processes in the one-dimensional space with known autocorrelation functions have been simulated. These simulated data sets were used for the experiments.

**ESTIMATION OF AUTOCORRELATION**

Let $Z(t)$ be a stationary random function in the one dimensional space with the autocorrelation function $B(s)$ defined as:

$$B(s) = \frac{E[Z(t) - m](Z(t+s) - m)}{\sigma^2}$$

(1)

where $m = E(Z(t))$ and $\sigma^2 = \text{var}(Z(t)) = E(Z(t) - m)^2$. $B(s)$ can be interpreted as the population correlation coefficient of the pair of random variables $(Z(t), Z(t+s))$.

When $z(t)$ is an observed value of $Z(t)$, $z(t)$ is called a realization of $Z(t)$. In practice, only a few $z(t)$ at discrete points $t = t_1, t_2, \ldots, t_n$, for $Z(t)$, are usually