FUZZY OPTIMIZATION AND MATHEMATICAL PROGRAMMING: A BRIEF INTRODUCTION AND SURVEY

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Abstract. Some general concepts and ideas related to fuzzy optimization as, e.g., a fuzzy constraint, fuzzy goal (objective function), fuzzy optimum, etc. are introduced first. A general fuzzy optimization problem involving these elements is formulated and solved. The cases of single and multiple objective functions are dealt with. Secondly, basic classes of fuzzy mathematical programming are discussed, including: fuzzy linear programming (with single and multiple objective functions), fuzzy integer programming, fuzzy 0-1 programming and fuzzy dynamic programming. Finally some newer, knowledge-based approaches are mentioned. An extended list of literature is included.

Keywords: fuzzy decision making, fuzzy optimization, fuzzy mathematical programming.

1. INTRODUCTION

The book focuses on optimization problems which belong to a much wider class of decision making problems.

Decision making has always played, and is playing, a crucial role in human life. In fact, any human activity is a succession of decision-making-related acts. A growing complexity of social, economic, technical, military, etc. problems faced by human decisionmakers has finally led to a necessity of using some formal (scientific) tools. This has stimulated the development of modern mathematical tools and techniques for that purpose.

The analysis of a real decision making situation is virtually based on two types of information:

- information on feasible alternative decisions (options, choices, alternatives, variants, ...),
- information making possible the comparison of alternative decisions with each other in terms of "better", "worse",

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"indifferent", etc.

To apply mathematical tools and techniques, these two types of information should be adequately quantified and formalized in the form of some mathematical models.

Having such models, diverse, more or less formal analytical methods may be used by the analyst to derive a rational choice (s) to be recommended to the decision maker. Evidently, the applicability of analytical methods at the analyst's disposal depends in a straightforward way upon the form of the model employed to represent a real decision making situation.

If a model of a decision making situation is not adequate enough, then the results of analysis may be misleading. This may also occur in case of unreliable or inaccurate data. Unfortunately, in many cases - above all in economic, social etc. systems where human judgments, preferences, etc. play a crucial role - information on a particular decision making situation must be elicited from human experts. It is therefore full of subjectivity and of ambiguity or vagueness which stem from the use of a natural language that is the only fully natural means of human communication. And it is our ability to adequately incorporate this type of information in an analytical mathematical framework that is crucial for enhancing the applicability of mathematical methods in real-world decision making situations.

Optimization problems constitute a wide class within decision making. Basically, information on the preferences among alternatives is in them described by some utility (objective, performance, ...) function that maps a given set of feasible alternatives into the real axis; this allows one to compare the alternatives with each other in a straightforward way through their numerical evaluations as, e.g., the greater the value of that function, the better the corresponding alternative.

The set of feasible alternatives in an optimization problem is frequently described by a system of equations and/or inequalities. In such a case the problem is referred to as one of mathematical programming.

Methods and techniques of optimization, or - more specifically - those of mathematical programming have been successfully used for years in various problems involving, and related to, technical systems of relatively well-defined structure and behavior, the so-called "hard" ones. This has allowed the formulation of corresponding optimization problems with precisely specified constraints and objective functions solvable by well-developed and quite efficient traditional analytical and computational means.

That success has motivated a direct application of the same traditional approaches to the modeling and analysis of what is often called the "soft" systems in which a key role is played by human judgments, preferences, etc. Unfortunately, the progress in this direction has been much less than expected, which has even raised doubts whether traditional mathematical tools are at all applicable to problems with relevant human-related elements.

It seems, however, that a more justified viewpoint is pro-