CHAIRMAN PARADOXES UNDER APPROVAL VOTING*

1. INTRODUCTION

In his pioneering analysis of sophisticated voting, Farquharson (1969) demonstrated that a chairman with both a regular and tie-breaking vote may do worse under the plurality procedure than if he had only a regular vote (no tie-breaker paradox). In a recent paper, we (Brams, Felsenthal, and Maoz, 1986) demonstrated that two related paradoxes can occur. Specifically, a chairman may do worse

(1) having both a regular and tie-breaking vote versus having only a tie-breaking vote (no-vote paradox);
(2) having a tie-breaking vote and participating and using it versus not participating in the election at all (no-show paradox).

We argued that these consequences of sophisticated voting are paradoxical because a chairman in each case has ostensibly greater power (a tie-breaking vote in Farquharson’s no tie-breaker paradox and the no-show paradox, a regular vote in the no-vote paradox) that he would better do without or not use.

In the case of the no-vote paradox and the no-show paradox, we posit a chairman with only a tie-breaking vote and compare the resulting outcome with what the chairman would have achieved if he (or she) had either an additional regular vote or been a nonparticipant rather than a participant. The role of tie-breaker is, in fact, the normal role of the chairman. For example, the U.S. Vice President, in his position as President of the U.S. Senate, can break ties but does not have a regular vote. We know of no voting bodies, at least in the United States, in which a chairman has both a regular vote and a tie-breaker, as is assumed in Farquharson's no tie-breaker paradox.

Our purpose in this paper is to investigate the incidence of the three chairman paradoxes under approval voting (Brams and Fishburn, 1983), whereby voters can cast one vote for each of the alternatives or candidates they approve of or find acceptable in a multicandidate
election (one with more than two candidates). We shall compare these results with plurality voting, whereby a voter can vote for only one candidate.

Our comparison will be limited to the most common form of determining winners in elections — namely, the plurality procedure — under which the candidate with the most votes (plurality or approval) wins. The plurality procedure, which describes not only how the winner is determined (the candidate with the most votes) but also how the candidates are voted upon (all at once rather than in stages), should be distinguished from plurality voting, which is the method of balloting that restricts a voter to casting only one vote in a multicandidate election. Sophisticated voting, by contrast, assumes that voters vote strategically in a noncooperative game of complete information in which dominated strategies are successively eliminated; we shall illustrate its calculations in several examples that manifest the chairman paradoxes under approval voting.

2. PARADOX LOST AND PARADOX REGAINED

Farquharson's (1969, 50—51) original example of the chairman paradox posits three voters with cyclical preferences abc, bca, and cab among three alternatives or candidates a, b and c, so there is no Condorcet candidate who can defeat the other two candidates in pairwise contests. (Neither this nor the subsequent paradoxes depend on the existence of cyclical preferences.) If abc is the chairman, with both a regular vote and a tie-breaking vote, the sophisticated outcome, under plurality voting, is c, the chairman's worst outcome.

Under approval voting, a voter would choose between two admissible, or undominated, strategies: vote for his best, or his two best, candidates. Because any other strategy, which necessarily involves voting for his worst choice or not voting for his best, is dominated, we need consider only the two admissible strategies of each voter shown in Figure 1.

The outcomes that can arise from each voter's voting for one or both of his two preferred candidates are also shown in Figure 1, where a slash between or among two or three candidates indicates a tie. If abc is chairman, these ties will be broken in favor of the parenthetic outcomes shown in this figure.

Comparing the first column b and the third column bc outcomes, the