Chapter 12

Ray tracing for surface waves

N. Jobert and G. Jobert

1. Introduction

Surface waves (at least the fundamental mode, see below) are the latest arrivals on the seismic record at a given epicentral distance. On classical records, they show up as long-period oscillations, predominating in the record for shallow earthquakes and distances larger than a thousand kilometers. Their relative importance increases with the distance, which can be explained by their 2-dimensional propagation in a direction parallel to the surface of the Earth. Indeed they are guided waves with standing wave properties in a direction normal to this surface: along this direction the displacements are in phase at all depths. Thus their geometrical spreading, as guided waves, is less than that of body waves which propagate in 3 dimensions.

Surface waves are classified in two types: the faster Love wave with a displacement transverse (y-component) to the direction of propagation (x-component), and the Rayleigh wave with a displacement in a vertical plane containing the direction of propagation (x and z components). Their theory has been known since 1885 for the Rayleigh wave, 1910 for the Love wave. At the surface of the Earth they behave as waves guided in the crust and upper mantle where the velocities are lower than at depth. Love waves are built with transverse SH waves multiply reflected at the surface of the Earth, Rayleigh waves with multiply-reflected and converted P and SV waves. Thus their local velocities are respectively related to the local velocity structure of these body waves. But it can be shown that except for the short wave-lengths influenced by the very shallow layers, the fundamental mode of Rayleigh waves is sensitive mainly to SV wave velocity.
For a guided wave, the theory shows the existence of an infinity of modes characterized by an index $m$. At a given frequency their phase velocities increase with $m$, and the variation of their displacement with depth shows a number of "nodes" related to $m$. The fundamental mode corresponds to $m=0$. For a horizontally layered half-space the frequency range of existence of higher modes ($m>0$) is more and more restricted to high frequencies by a cut-off frequency increasing with $m$. But the fast higher modes are less excited by shallow earthquakes than the fundamental mode which is generally well observed.

As long as the guide which supports them does not present a strong lateral variation with distance, surface waves can be used to retrieve average structural information at depth between two stations (or source and station) which can be widely separated. Hence their efficiency to study oceans or inaccessible regions, as long as the structure remains not far from laterally homogeneous between the two points. The different frequencies present in the records have only to be analyzed and separated. Due to dispersion, which is a property characteristic of surface waves, each monochromatic wave propagates with its own velocity. The fundamental mode behaviour is such that each of them is most sensitive to the structure at a depth which is a fraction of the wave-length, about $1/3$ for Rayleigh waves, $1/4$ for Love waves (Knopoff, 1972). Consequently, the resolution of the inverse problem for the variation of velocity structure with depth is all the better as the bandwidth of data is broader. However, compared to that of body waves, the resolution is limited by the long-period character of the surface waves and informations are averaged over a certain depth interval. The maximum depth down to which information is obtained is independent of the distance between stations, so that large depths can be reached with a few long-period stations. Surface or mantle waves carry also along their path long-period information about the source.

Quantitative information began to be obtained with the development of long-period instruments (Press, 1956, Sato, 1958). A first group velocity tomography of the surface of the Earth was presented by Santo (1966). Since 1960, the instrumental techniques have greatly improved. Now to seismologists are available high-quality long-period data, digitally recorded in the stations of arrays such as GDSN, IDA or GEOSCOPE. The precision of measurements increased, and long-period tomography has developed to bring out large-scale laterally heterogeneous models of the Earth, such as that by Nataf, Nakanishi et Anderson (1986), obtained by inversion of velocity measurements, or M84C (Woodhouse et Dziewonski, 1984) obtained by waveform modelling. These new results show large scale structure anomalies in the mantle that can be related to convection patterns.

In the classical methods of surface wave tomography, the waves are supposed to propagate around the Earth along great circles. Considering Fermat’s principle, this is valid in the geometrical optics approximation, and first order perturbation theory, for smooth and weak lateral heterogeneities. In the great circle tomographic technique, a regionalization is performed to retrieve regional "pure-path" velocities in regions considered as laterally homogeneous. The slowness observed along a path is considered to be the sum along the great circle path of the local slownesses weighted by the relative lengths of the corresponding "pure paths". This is an expression for finite lengths of great circle paths of