STOCHASTIC FORMULATION OF STORM PATTERN AND RAINFALL INTENSITY-DURATION CURVE FOR DESIGN FLOOD

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ABSTRACT. A stochastic single storm pattern, which preserves stochastic properties of actual storm rainfall, is theoretically derived from Freund's bivariate probability density function. Two typical design storm patterns, namely, the last and the central peaked types are defined by the three parameters: the reduced variate $y_p$ of the peak rainfall intensity, the autocorrelation index $k$ related to the autocorrelation coefficient of the rainfall intensities, and the conditional probability $F$. Integration of the given stochastic design hyetograph gives a new 'conditional probability' intensity-duration formula. Furthermore, a practical estimation method of the three parameters: $k$, $F$, and $y_p$ is clearly shown. The conditional probability intensity-duration curve is demonstrated by using actual hourly rainfall data. Design intensities for shorter duration than 1 hr can be easily estimated from the available hourly data.

1. INTRODUCTION

In order to estimate the magnitudes of design floods in urban and rural areas, many designers of drainage systems have used the concept of a single design storm which is used in conjunction with an appropriate runoff model for the objective drainage system. The time distribution of rainfall intensities is determined from a rainfall intensity-duration-frequency relationship in the form:

$$ r = a / ( t^b + c ) \tag{1} $$

where $r$ = average rainfall intensity for duration $t$ and $a$, $b$, $c$ = empirical constants depending upon return period. Although several forms including Eq. 1 were empirically proposed, they have been hardly supported by the theory of probability that represents stochastic properties of actual storm patterns.

The important objections to the use of the design storms determined from the empirical intensity-duration formulas is that they attempt to summarize 'various' storm patterns in a single hyetograph.
shape and assemble components of equal individual probability of occurrence which have a quite different joint probability (Delleur and Dendrou, 1980). Frederick (1978) demonstrated that the joint occurrence of large rainfall totals within different durations of the same storm had a lower probability than the rainfall intensity-duration curve from which they were derived.

In this paper, a stochastic single storm pattern, which preserves stochastic properties of actual storm hyetograph, is theoretically derived from the bivariate probability density function defined by Freund (1961). Integration of the stochastic single storm hyetograph gives a new 'conditional probability' intensity-duration curve. The stochastic storm pattern is prescribed by three parameters: the reduced variate \( y_p \) of the peak rainfall intensity, the autocorrelation index \( k \), and the conditional probability \( F \) that represents the conditional probability of a rainfall intensity \( y_i \) given the advanced intensity \( y_{i-1} \). A practical estimation method of the three pattern parameters is presented, and the conditional probability intensity-duration curve is demonstrated by using actual hourly data.

2. STOCHASTIC FORMULATION OF STORM PATTERN

2.1. A Single Storm Pattern Represented by Freund's Distribution

Freund (1961) defined a bivariate exponential density function \( f(\xi, \eta) \) of a bivariate \((\xi, \eta)\) with four parameters: \( \alpha_1, \beta_1, \alpha_2, \) and \( \beta_2 (>0) \). For the case \( \alpha_1=\beta_1=\alpha \) and \( \alpha_2=\beta_2=\beta \), the variables \( \xi \) and \( \eta \) have identical marginal distributions. Consider a single storm pattern with a peak of rainfall intensity \( x_p \) as shown in Fig. 1.

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\begin{align*}
\text{Fig. 1 A single storm pattern with a peak}
\end{align*}
\]

Let the intensities before and after time unit \( \Delta t \) be \( x_{i-1} \) and \( x_i \), respectively. By putting \( \xi=x_i \) and \( \eta=x_{i-1} \) and assuming that \( x_i \) and \( x_{i-1} \) have identical marginal distributions, we obtain the following density function.