dealing with the person's preference for an apple over a pear and over a nut, we used this type of representation without considering for a moment what his motive was. Motive is one of the aspects of choice which is abstracted by Economics and similarly it can be abstracted in the theory of committees. This particular objection is not likely to give the slightest trouble to anyone acquainted with Economics.

The objection (d), that different people expect quite different consequences to result from the adoption of the motion \( a_1 \), say, is in no better case. In Economics we do not ask what consequences the person expects to result from eating the apple; and it is extraneous to the theory of committees to ask what a person's expectations are in regard to the consequences of the adoption of any motion.

These are the main objections which may be brought against the symbolism we use. Certainly they raise difficult questions in the theory of knowledge, as indeed do all similar queries that may arise in any branch of mathematical science. But there seems to be no doubt that the symbolism we use is valid.

CHAPTER IV

A COMMITTEE USING A SIMPLE MAJORITY: SINGLE-PEAKED PREFERENCE CURVES

A quite general theorem. The following will hold whatever the shapes of the members' preference curves may be:

**Theorem.** *At most, only a single motion can get a majority over every other.*

*Let us assume that two motions, \( a_h \) and \( a_k \) say, can each get a simple majority over every other. Then \( a_h \) can get a simple majority over \( a_k \) and \( a_k \) can get a simple majority over \( a_h \), which is absurd. Hence at most only one motion can get a simple majority over every other.*

*An illustration of the subsequent argument.* We will assume that in a committee \( m \) motions are put forward (\( m \) finite or infinite) and that an ordering can be found for the points to represent

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D. Black, *The Theory of Committees and Elections*  
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these motions on the horizontal axis, such that the members' preference curves become single-peaked.

To show the method of reasoning employed we will give an arithmetical example. Fig. 12 shows the preference curves of the five members of a committee. Only part of each curve has been drawn, and the curves are supposed to extend over a common range of the horizontal axis. We name the members' optima $O_1, O_2, \ldots$, in the order of their occurrence as we move from left to right along the horizontal axis.

Then if $a_h$ is put against $a_k$ (where $a_h < a_k \leq O_1$), the preference curve of each member—irrespective of what its precise shape may be—is up-sloping from $a_h$ to $a_k$; and $a_k$, standing at a higher level of preference on the curve of each member, will get a 5:5 (5 out of 5) majority against $a_h$. If $a_h$ is put against $a_k$ (where $a_h < a_k \leq O_2$), at least four members—viz. those with optima at or above $O_2$—will have preference curves which are up-sloping from $a_h$ to $a_k$; and $a_k$ will get at least a 4:5 majority against $a_h$. If $a_h < a_k \leq O_3$, $a_k$ will get at least a 3:5 majority against $a_h$. And similar relations hold for motions corresponding to values above $O_3$. If two values above $O_3$ are placed against each other in a vote, the nearer of the two values to $O_3$ will get a majority of at least 3:5 against the other.

If a value $a_h$ (where $a_h < O_3$) is put against a value $a_k$ (where $a_k > O_3$), before we could find which of the values would win in a vote, we would have to draw the complete preference curve for each member, find whether $a_h$ or $a_k$ stood higher on the preference curve of each member and count up the votes cast for $a_h$ and $a_k$. But even though a value below the median optimum $O_3$ should defeat all values to the left of itself, and should defeat some of the values above $O_3$, this would be without significance, because what