RESPONSE OF SPHERICAL SHELLS UNDER APEX LOAD TO VARYING PROPORTIONS

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1. ABSTRACT
Two thin spherical closed segments of equal base-diameters, with a low and a high rises, were selected as testing objects. A uniform distributed load over a small circle at the apex and pinned-edge-conditions have been considered. Spherical segments are investigated analytically by the force method, numerically by implementing the finite element method and experimentally by means of electrical resistance strain gages. Apex loads inside the shallow zone seem to require special considerations.

2. INTRODUCTION
The characteristics of spherical caps depend, among other factors on the ratio of rise to diameter of the outer-most parallel circle. This ratio will be referred to hereafter as the arching parameter (AP). The performance of spherical shells as structural elements varies, in terms of their AP, from that of a circular plate at one extreme to that of a spherical shell at the other. Shallow spherical shells whose geometrical properties eliminate some terms of the general solution of spherical shells, represent an intermediate stage.

Theoretical analyses of spherical shells cited in literature usually either the case of clamped edges or rolling supports along the outer-most parallel circle. A uniform distributed load over the entire surface-area is a common case of loading. Herein, spherical shells with hinged boundary conditions and apex load within the shallow zone are considered. These are of practical importance in the area of civil engineering. Besides, the validity of existing approaches under the above mentioned case of loading seems to require further study.

The present work investigates the surface stresses of two spherical shells whose AP are selected so apart to provide a shallow and a nonshallow spherical segments. Surface-stress-distribution is presented as a criterion for assessing the characteristics of both shells as well as the approaches employed.

Spherical shells are defined by the radius of the sphere, R, the rise, h, and the diameter of the outer-most parallel circle, 2a₀ (Fig. 1), known also as the base diameter. In general, theoretical solutions of spherical shells involve analytic functions, power series or periodic functions [1-9]. Several theoretical approaches are based on establishing differential equations of the second order whose solutions result in the so-called stress and displacement functions [1-3]. These functions contain arbitrary constants that could be determined from the boundary and transition conditions. The "force method" is adopted herein due to its flexibility in modifying the solution to match with the present boundary conditions.

The scope of analysis is widened to include a numerical approach, namely the finite element method. A simple computer program is prepared to

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generate the required data for establishing the finite element grid of any spherical shell. This makes finite element analysis convenient, if proved to reflect the true response of spherical segments under the present conditions. Based on a slice of the spherical cap that is limited by two meridional planes which form between them a five-degrees-dihedral angle, and the outer-most parallel circle, as shown in Fig. 2, the finite element grid (a gore) is defined. The package SAP V was invoked in this analysis. The possible rotation of the finite element about the normal to its surface is taken into account. In the case of shallow shells, this rotation becomes a significant problem. The selected gore along with the boundary elements, described below, provide a finite element grid that can be implemented for spherical segments of different APs.

The analytical and numerical results are checked experimentally. For this purpose, spherical segments supplemented with a sufficient number of measuring stations were considered. Components of each measuring station and their arrangement within the station as well as the selection of their locations on both surfaces of the spherical segment were designed to provide a thorough scanning of surface stresses. The applicability and particularities of each approach are pointed out. The results obtained from the three approaches are compared and discussed.

3. MATHEMATICAL FORMULATIONS

In general, exact theoretical solutions for thin shells can be achieved by means of superimposing the results obtained from the membrane and bending theories. The final results are worked out in two steps. First the membrane forces are determined for the boundary conditions shown in Fig. 3a. This solution assumes unrestrained deformation along the edge of the outer-most circle of the spherical shell under consideration. In a further step, the flexibility coefficients at the edge of the spherical shell are determined. These are the torsional flexibility about the tangent to the outer-most parallel circle and the translational flexibilities in both horizontal and vertical directions. This leads to the formulation of the edge deformation due to unit edge actions, M, H and V as shown in Fig. 3b. The actual values of M, H and V are obtained by satisfying the existing boundary conditions. This approach is known as the force method [7].

Spherical segments are shells of revolution subjected, in the present case, to an axisymmetrical load. It follows that the internal forces