Introduction.

Quantum field theory combines the principles of quantum mechanics with the principles of special relativity and with the principles of causality and locality. Such a combination leads to an infinite number of degrees of freedom conveniently labeled e.g. by the points of space. The wave-function is replaced by a wave-functional supplying a probability amplitude for a field configuration. The last fifteen years brought about an understanding of quantum field theory in general and non-abelian gauge theories in particular that goes beyond the understanding gained through the application of simple perturbation theory. The most important achievement of this development is the so-called "standard model" of particle physics. As part of this development it was realized that there are tunneling phenomena in quantum field theory. It is thus not surprising that more than a decade ago a calculational scheme known as the "dilute instanton gas approximation" was developed by A.M. Polyakov and by G.t'Hooft to deal with tunneling phenomena.

The purpose of this talk is to review this scheme using two simple one dimensional quantum mechanical problems as examples and trying to emphasize the aspects of quantum barrier penetration. The moral of the story is that a practitioner of field theory is interested in a calculational scheme that is generalizable to any number of degrees of freedom and in particular an infinite number of degrees of freedom. It should be said explicitly that this generalizability to field theory is not an interesting property as far as the practitioner of quantum mechanics is concerned. On the contrary, the examples that I shall present may be dealt with more directly and the method I shall review may look cumbersome in the context of these examples.

The material presented is neither new nor original and it is presented for the purpose of facilitating communication among scientists from various disciplines.
The Double Well Potential.

Let us start with a simple quantum mechanical system of a non-relativistic particle of mass $m$ moving on the line $-\infty < q < \infty$ under the influence of a non negative potential $V(q)$. The Hamiltonian of the system is given by

$$H = \frac{p^2}{2m} + V(q)$$

We further assume that if for a given energy there exist several disconnected regions of allowed classical motion they are related to each other by a symmetry of the Hamiltonian $H$. This means that if $H$ has a minimum which is not invariant under $G$ -the group of symmetries of $H$- this minimum must be energy degenerate with other minima of $H$ so that the set of minima is invariant under the symmetry group $G$. Let us review two simple examples.

I. consider the potential $V(q) = \frac{1}{2} m \omega^2 q^2$

The symmetry group of the Hamiltonian is the two element group $Z_2$ generated by reflections at the origin or parity

$$P : q \rightarrow -q$$

Clearly the only invariant point is the origin $q = 0$. It is also clear that the potential $V$ is parity invariant. Now $V$ has a single minimum at the invariant point and the Hamiltonian has a single classical ground state namely $p = 0$, $q = 0$. This is the state of a particle sitting at rest at the bottom of the well.

II. The second example involves the potential function $V(q) = \frac{\lambda}{4} \left( q^2 - \frac{\mu^2}{\lambda} \right)^2$ with $\lambda > 0$ and $\mu > 0$.

The symmetry group is again $Z_2$ generated by parity. In this example the potential has two minima located at $q = \pm \sqrt{\frac{\mu^2}{\lambda}}$ and transforming into each other under parity. Thus there are two classical ground states corresponding to the particle being at rest at the bottom of either the left well or the right well. Clearly any classical motion with small enough energy breaks the left-right symmetry as it takes place in only one of the two wells.

Quantization brings with it the possibility of tunnelling. It is clear that in this simple example the system will indeed tunnel and the two lowest eigenstates of the Hamiltonian $H$ will have well defined parities. The question that needs an answer in the most general situation is similarly: does the system tunnel between degenerate ground states? or stated with an emphasis on the symmetry...