NUMERICAL METHODS IN FRACTURE MECHANICS

W. Schmitt
Fraunhofer-Institut für Werkstoffmechanik
Wöhlerstrasse 11
7800 Freiburg
Federal Republic of Germany

ABSTRACT. After a short introduction sketching the continuum mechanics description, the basic equations of Fracture Mechanics will be evaluated with particular emphasis on the numerical treatment.

Among the numerical methods the Finite Element Method (FEM) will be discussed in more detail, including special crack tip elements, energy release rates and path-independent integrals.

The problem of short cracks will be touched as well as the damage function approach. A series of actual cases will be presented to demonstrate the applicability and validity of the methods introduced.

1. Introduction

The mechanical behavior of solids may be understood on the basis of the atomistic structure of the material and of the forces which cause the coherence of individual atoms or molecules. The macroscopic strength expressed e.g. as the maximum stress that can be sustained by a structural member of this material is only to a small extent influenced by theoretical atomic or molecule bond forces, as discussed by Sommer in the first lecture of this seminar. Instead, the characteristics of imperfections, their number and distribution determine the actual resistance of a material against failure.

Since information about these microscopic quantities is generally not readily available, and, if available, it could not directly be used to determine the actual behavior of a structure, as a first approximation to the real problem the concept of a continuum has been introduced. The continuum can be regarded as the mechanical model. The state of a continuous body can be described in terms of stresses, strains and displacements. Between those quantities mathematical relations exist in the form of e.g. differential equations. These relations already define the mathematical model, which also includes the boundary conditions defining an actual problem. Since exact solutions are available only for a very limited number of mathematical problems, in general numerical models have to be set up and solved.

It becomes now evident that this chain of simplifications from the real problem down to a mathematical or even numerical model must be
regarded as a source of systematic errors. This may be demonstrated by
the example of an elastic-plastic fracture mechanics test with a compact
specimen which was to be analysed by the finite element method. Figure 1
demonstrates a very good agreement between analysis and experiment
(here: force-displacement curve, from [1]) if a 3D-analysis is con-
sidered. Attempts to apply 2D-models lead only to some qualitative
agreement in the overall performance of test and analysis. Two-dimen-
sional simplifications in numerical analysis have been employed mainly
in order to reduce the computational effort. This argument becomes less
important with the rapid increase in available computer power.

In analytical and theoretical work the reduction by one dimension
allows the study of singular stress and strain fields in the vicinity of
crack tips resulting in a variety of near-field solutions. It must be
emphasized, however, that mechanical or mathematical models incorpo-
rating only a reduced number of dimensions represent a significant de-
viation from the (three-dimensional) reality. This principal deficiency
cannot be overcome by any numerical effort. The numerical solution will
not be closer to reality than the mechanical model, except by error.

Fig. 1: Compact specimen, force-displacement diagrams, ref. 1