CONFIDENCE INTERVALS FOR RIDGE REGRESSION PARAMETERS

ABSTRACT

This paper reviews various alternatives for constructing confidence intervals for ridge regression (RR) parameters, and illustrates them with an example. Among the newer alternatives are bootstrapping and those based on Stein's (1981) unbiased estimate of the mean squared error (MSE) of a biased estimator of multivariate normal mean. A simulation study supports the validity of the confidence statements based on Stein's model as modified here for the ridge regression problem. It yields confidence intervals which can be more useful and reliable than those based on other methods.

1. INTRODUCTION

Although ridge regression has been used in econometrics by several authors, there is an impression among some econometricians that reliable confidence intervals are unavailable. The available alternatives offer a trade off between computational expense and power. This paper provides a comprehensive discussion, currently unavailable in the literature, of these alternatives. A new alternative based on the methods of Stein (1981) is discussed.

The simplest alternative advocated by Obenchain (1977) is to use confidence intervals based on ordinary least squares (OLS), even if one uses the point estimates from ridge regression. Unfortunately, the OLS interval is sometimes meaningless on a priori grounds. For example, Vinod and Ullah (1981, p. 12) gave an illustration of an OLS estimate of a consumption function where the OLS estimate of the marginal propensity to consume out of wage income is −0.17, thus having the wrong sign; a corresponding 95% confidence interval is (−0.26, −0.8), each point of which is invalid for substantive reasons. Such an OLS interval is centered at the wrong point; the OLS
value of \(-0.17\) and is economically meaningless. When such problems arise the best strategy is to look for better specifications. However, there are cases where, within the range of specifications consistent with economic theory, none yields reasonable OLS estimates. This is where RR is an attractive alternative to OLS, and where much of the OLS confidence interval theory may be economically meaningless. For examples where the OLS interval is meaningful, the practitioner need not consider other alternatives.

A second alternative is the approximate Bayes method, discussed near the end of Section 3. The interval is centered on the RR coefficients, rather than those for OLS, and the standard errors are based on the diagonals of the inverse of the \((X'X+kI)\) matrix in the usual notation which is defined below. Both Bayesians and frequentists can find philosophical or other reasons for rejecting this interval. In my opinion, approximate Bayes method offers a quick and reasonable alternative as a first approximation, provided the practitioner is willing to assume that RR is appropriate for the estimation problem and overlook philosophical questions.

A purpose of this paper is to propose a third alternative, namely a frequentist confidence interval based on Stein's (1981) use of the unbiased estimate of the MSE (UMSE) of biased estimators. If \(x\) is a normal random variable with mean \(\xi\), its biased estimator is \(\delta x\), where \(\delta\) is a shrinkage factor. Stein's confidence interval based on a property of normal variables is discussed in Section 2, along with some modifications. In the remaining part of Section 1 we will develop notation, similar to Vinod and Ullah (1981), such that the ridge estimator can be thought of as a shrinkage factor \(\delta\) times a normally distributed variable, denoted by \(C\) in (1.7) below. Section 3 explicitly applies the methods in Section 2 to the construction of confidence interval for RR; these intervals are illustrated by Hald's (1952) cement data in Section 4. Section 5 discusses a simulation study based on the data structure in Hald's cement data to assess the validity of confidence statements. An appendix discusses the case of stochastic \(k\).

A fourth alternative is to use bootstrap resampling (Efron, 1982) to construct a sampling distribution. Efron's "bias corrected percentile method" can then be used to construct confidence intervals. Section 6 explains the mechanics of using the bootstrap for RR, illustrates the method with the cement data, and also discusses the "qualms" that Schenker (1985) associated with using the bootstrap intervals. We note that bootstrap intervals may be useful for understanding the sampling distribution, provided we already know that RR is a good estimate of the unknown parameter.

Let us consider the general linear regression model in the standardized form as:

\[
y = X\beta + u, \quad E(u) = 0, \quad E(uu') = \sigma^2 I_n, \quad (1.1)
\]