6 GROUP MACHINE LOADING WITH VARIABLE PROCESSING TIMES

6.1 Basic Models of Production Systems with Variable Processing Times

In most cases in the field of production planning and scheduling, the processing time required to complete a specified operation of a job is set as a constant. In the previous chapter, the machine loading models were constructed under the assumption that the processing time is a constant. In practical situations, however, it is possible to vary the processing times by actively changing manufacturing conditions, especially machining speeds. In these cases, some modifications must be made to the production planning and scheduling models. In order to solve those models, a new type of analysis must be made allowing for variation in processing times and costs. This chapter treats the machine loading models with variable processing times. First, a basic production model with variable processing times is covered, and then it is extended to the machine loading models.

A basic model of production systems with variable processing times is as follows [1]:

1. Unit production time. This is the time needed to manufacture a unit piece of a product. This unit production time \( u(\text{min/pc}) \), is assumed to be composed of the preparation time (or setup time), machining time, and tool-
replacement time, and is expressed as a function of machining speed, \( v(\text{m/min}) \), as follows:

\[
    u = a + t + b \cdot \frac{t}{T} \\
    = a + \frac{\lambda}{v} + \frac{\lambda b}{C^n} v^{\frac{1}{n} - 1}
\]

(6.1)

where \( a \) is the preparation time (min/pc), \( t \) is actual machining time, \( b \) is tool replacement time (min/edge), \( T \) is the tool life (min/edge), \( \lambda \) is the machining constant, and \( n \) and \( C \) are parameters for the Taylor tool-life equation.

The time curve with regard to machining speed given by equation 6.1 has a minimal point \( v(t) \), which is called the “maximum-production-rate or minimum-production-time machining speed,” and is determined by setting the derivative of equation 6.1 to \( v \) equals zero:

\[
    v(t) = C \left[ \left( \frac{1}{n} - 1 \right) b \right]^n
\]

(6.2)

2. Unit production cost. This is the cost required to manufacture a unit piece of product. This cost, \( q(\$/pc) \), consists of the preparation cost, machining cost, and tool-replacement cost, and is expressed as a function of machining speed as follows:

\[
    q = \alpha a + (\alpha + \beta) t + (\alpha b + \gamma) \frac{t}{T} \\
    = \alpha a + (\alpha + \beta) \frac{\lambda}{v} + (\alpha b + \gamma) \frac{\lambda}{C^n} v^{\frac{1}{n} - 1}
\]

(6.3)

where \( \alpha \) is the direct labor cost and overhead (\$/min), \( \beta \) is the machining overhead (\$/min), and \( \gamma \) is the tool cost (\$/edge).

The cost curve with regard to machining speed given by equation 6.3 has a minimal point \( v(c) \), which is called the “minimum-production-cost machining speed” and is also determined by setting the derivative of equation 6.3 to \( v \) equals zero:

\[
    v(c) = C \left[ \left( \frac{n}{1 - n} \right) \left( \frac{\alpha + \beta}{\alpha b + \gamma} \right) \right]^n
\]

(6.4)

3. Unit profit. This is the gain obtained by producing a unit piece of product. This profit is the unit revenue or selling price minus unit production cost: