Sinusoidal Carrier Modulation

Objectives

- To describe how modulation enables a message to be matched to a band pass channel.
- To show that amplitude modulation corresponds to multiplication of a message with a sinusoidal carrier.
- To determine the spectral occupancy of various amplitude modulated signals (DSB-SC, SSB-SC, conventional envelope modulation) in terms of the carrier and message spectrum.
- To explain how a message is recoverable from an AM signal using coherent detection.
- To assess the influence of phase errors on coherent detection.
- To show that angle (frequency or phase) modulation can be used to convey a message over a band pass channel.
- To determine the spectral occupancy of a composite signal produced by frequency division multiplexing.

Introduction

The reader may be aware that radio systems generally use rather high frequencies. For example, the so-called ‘citizen’s band’ is in the vicinity of 27 MHz. On the other hand, it has already been seen that speech signals are concentrated at relatively low frequencies, $|f| < 20$ kHz. How then can speech signals be conveyed over a radio channel? The answer is that a process called modulation is used to match the speech to the available communication channel.

This process of modulation involves the variation of some parameter of one signal (the carrier) by another (the message), the result being a modulated carrier. The modulated carrier contains complete information about the message signal and the original message can be recovered by suitable signal processing.

There are various forms of modulation. Consider a sinusoidal signal as the carrier: an information-bearing message signal can be impressed on to this by varying the amplitude, the frequency or the phase. All of these schemes are used, both separately and in combination. Only the basic methods are examined here, paying particular attention to the simplest, namely amplitude modulation.

Amplitude Modulation

There are various forms of amplitude modulation; only three are considered here: (i) double sideband suppressed carrier modulation, (ii) single sideband modulation, and (iii) conventional (envelope) amplitude modulation.

Double Sideband Suppressed Carrier Modulation

Let $m(t) = A_m \cos \omega_m t$ represent a message signal and $x_c(t) = A_c \cos \omega_c t$ a carrier, with...
A multiplication process is at the heart of all amplitude modulation systems.

Recall that this representation in terms of 'positive' and 'negative' frequency components is largely a matter of mathematical convenience, a consequence of expansion in terms of complex exponentials.

If the two signals are multiplied together as shown in Fig. 3.1, the following is obtained:

\[ x(t) = m(t)x_c(t) = A_m \cos \omega_m t A_c \cos \omega_c t \]

\[ = \frac{A_m A_c}{2} \left[ \cos(\omega_c + \omega_m) t + \cos(\omega_c - \omega_m) t \right] \]  

This signal is concentrated in the vicinity of \( F_c \) and is composed of two terms known as sidebands. The term at \((F_c + F_m)\) is the upper sideband and the term at \((F_c - F_m)\) is the lower sideband. Notice that there is no component at the carrier frequency itself, hence the name double sideband-suppressed carrier (DSB–SC) modulation. The message information is carried in the sidebands. This modulation process can be interpreted in the frequency domain by writing the cosine waves in terms of phasors:

\[ x(t) = \frac{A_m A_c}{4} \left\{ \exp[j(\omega_c + \omega_m) t] + \exp[-j(\omega_c + \omega_m) t] \right. \]

\[ + \exp[j(\omega_c - \omega_m) t] + \exp[-j(\omega_c - \omega_m) t] \} \]  

This corresponds to a two-sided spectrum with positive frequency terms at \((\omega_c + \omega_m)\) and \((\omega_c - \omega_m)\) together with negative frequency terms at \(-(\omega_c + \omega_m)\) and \(-(\omega_c - \omega_m)\), as illustrated in Fig. 3.2.

With reference to these diagrams, note that the positive frequency part of the modulated signal is of the same form as the message spectrum but is centred on