THE STEP BEFORE THE APPLICATION OF FRACTURE MECHANICS TO FATIGUE

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ABSTRACT

The whole lifetime to fracture of a fatigue loaded part is divided into the crack initiation region and the crack propagation region. The crack propagation region can be treated by fracture mechanics methods. The Weibull theory is used to describe the scatter of crack initiation life and allows to consider the size effect. Test results from random loaded specimens show good agreement with the theoretical prediction.

INTRODUCTION

In fracture mechanics, the usual procedure to determine the lifetime of a cyclically loaded specimen starts with the assumption that a crack of a certain length \( a_i \) is inside the body. Then only two laws are needed to calculate the number of cycles to fracture \( N_f \):

- the crack propagation law \( \frac{da}{dN} = f(\Delta K) \) (1)
- the fracture criterion \( K_1 = \sigma \sqrt{a} Y \geq K_{IC} \) (2)

In the case of constant-amplitude-loading, \( N_f \) results from integration of equation (1), in the case of irregular loading the crack growth has to be computed step by step due to load interaction effects (retardation, acceleration).

In this formalism it is assumed that there is no scatter in lifetime, that equation (1) is valid over the whole range, and that the quantity \( K_{IC} \) (or a quantity somewhat lower) allows calculation of the critical crack length, which causes final failure.

CONSIDERATION OF THE SCATTER OF LIFETIME

With great success equation (1) was used by W. Ziebart [1] to determine a fictive original crack length \( a_0 \), which is present before constant-amplitude-loading. The statistical distribution of \( a_0 \), which is caused by the scatter
of $N_f$, can be used to describe the size-effect and to calculate $N_f$ for specimens of any shape. This method is based on the assumption that equation (1) is valid from the beginning of the loading which seems to be problematic especially at low-strength materials.

Several new publications dealing with the comparison of fatigue data gained from specimens, which are to be transferred to parts of another shape and size, avoid this difficulty by dividing the whole lifetime into two regions. Region I contains the nucleation and propagation of a crack to a detectable technical length $a_i$. This region is controlled by the statistical structure of the material. The scatter of the number of cycles $N_i$ to generate a crack of the technical crack length $a_i$ can be described by the Weibull-formula

$$P_i = 1 - \exp \left\{ \frac{S_i}{S_o} \left( \frac{N_i}{N_v} \right)^m \right\}$$  \hspace{1cm} (3)

where $P_i$ is the probability of technical-crack-initiation, $N_v$ and $m$ are constants, and $S/S_o$ is the magnification factor, if geometrically similar parts are compared.

The technical crack has to be of such length $a_i$ that further crack propagation can no longer be influenced by statistical effects caused by the structure. This leads directly to region II, where the number of cycles between $N_i$ and $N_f$ can be determined by fracture mechanics methods. It is assumed that the front of the technical crack is of the magnitude of about 100 to 1000 grain diameters and that the choice of this crack length within these bounds does not severely affect the accuracy of this procedure.

A summary of the procedure is shown in the following schematic diagram:

<table>
<thead>
<tr>
<th>Region I</th>
<th>Region II</th>
</tr>
</thead>
<tbody>
<tr>
<td>crack length</td>
<td>$0 \rightarrow a_i \rightarrow a_c$</td>
</tr>
<tr>
<td>range of number of cycles</td>
<td>$0 \rightarrow N_i \rightarrow N_f$</td>
</tr>
<tr>
<td>dominating effects</td>
<td>statistical to geometrical</td>
</tr>
<tr>
<td>theoretical treatment</td>
<td>statistical to fracture</td>
</tr>
<tr>
<td>equation (3)</td>
<td>equations (1) and (2)</td>
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</tbody>
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