SIZE EFFECTS IN SOLID MECHANICS DUE TO TOPOLOGY TRANSFORMATION DURING CRACK GROWTH

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ABSTRACT

A pseudo-elastic multi-damage analysis is carried out by utilizing the three-point bending test as structural geometry and the strain energy density theory as crack growth criterion. An elastic-softening constitutive law is considered and the damage of the material is obtained by decreasing elastic modulus and fracture toughness in relation to the strain energy density absorbed in the volume element. In this way, the strain energy density factor S results to be a linear function of the crack length a when both crack growth and structural behavior are in their respective stable stages. The S-a straight line shifts upwards when the size increases. The non-similarity in the mechanical and fracture behavior, when the structural size varies, is due to the different physical dimensions of two parameters involved in the analysis: the strain energy density and the strain energy density factor. When the structural size is higher than a certain limit, the stable crack growth stage is impossible and only the unstable and brittle crack propagation can occur. Vice versa, when the structural size is very small, the unstable crack growth ceases to occur and the stable propagation corresponds to a global collapse of a different nature (ultimate strength or plastic flow).

INTRODUCTION

It is well-known that over the last few years, a great effort has been made by research-workers and institutions all over the world in order to explain some peculiar and recurring phenomena in material and structural strength. Such phenomena can be basically summarized in two points:

1. geometry effects [1];
2. stable crack growth [2].

By "geometry effects", we denote a phenomenon according to which if structural geometry, initial crack configuration and/or structural size vary, the me-

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chanical behavior of the structure will decidedly change. While the first two effects can be easily understood and justified on the basis of traditional solid mechanics, the last one results to be of more difficult understanding and can be explained only by means of fracture mechanics and physical similitude concepts [3]. In fact, keeping the shape constant and varying only the scale, the structural behavior changes totally from brittle to ductile. In the same way, a structure is sensitive to the presence of a crack, i.e., there is an actual stress concentration - only above certain sizes, depending on the structure geometry, the crack geometry and the material properties [4].

By "stable crack growth", we denote a phenomenon according to which only a local instability occurs at the crack tip, thus producing a slow crack propagation. Its nature is completely different from the one of the unstable propagation. While the latter is a global or structural instability and may be predicted by the critical value \( S_c \) of the strain energy density factor [5] or by means of limit analysis concepts (depending on the scale), the former is a local or microstructural instability and depends on the critical value \( (dW/dV)_c \) of the strain energy density, which is the area under the stress-strain constitutive relationship. The fundamental laws ruling the transition from slow to fast propagation and vice versa, have not been clarified yet. They have to be very general laws applicable to both very simple and very complicated structures, so that it will be easy and consistent to extrapolate the results obtained from small specimens to the project of large and complex structures.

In the present work, the strain energy density theory is applied to a bilinear material with softening. The material damage is simulated by the degradation of both the elastic modulus \( E \) and the critical value \( (dW/dV)_c \) of strain energy density. The fundamental relationship is used [5,6] to compute the crack growth increment:

\[
\frac{(dW)}{dV}_c = \frac{S}{r} \tag{1}
\]

which connects the strain energy density factor \( S \) with the distance \( r \) from the crack tip.

By way of example, a three-point bend test specimen is studied by means of a finite element pseudo-elastic analysis and the damage at the crack tip as well as the slow crack growth prior to the global instability are analyzed step by step. The subsequent softening and unstable structural behavior is also analyzed; it can be preceded or not by the unstable crack propagation only depending on the scale. In fact, a dimensional analysis of the problem is carried out and the basic variables are reduced to a dimensionless form. It is proved, in this way, that the different physical dimensions of the two critical parameters involved in the analysis - \([(dW/dV)_c] = [F][L]^{-2} \) for local instability and \([S_c] = [F][L]^{-1} \) for global instability - together with the finiteness of the structural geometry, produce the well-known size effects of fracture mechanics. Two global collapses are always in competition in solid bodies: the traditional collapse due to the overcoming of structural strength and the global crack instability. By varying the specimen sizes, one of them prevails over the other.