1. INTRODUCTION

Over the last few years there has been an increasing interest in assessing the reliability of involved and expensive engineering structures and components being subjected to fatigue loading situations. The approach being taken in these studies has been and still is to describe, in a statistical manner, the microstructural material degradation processes taking place in polycrystalline materials and to infer from analytic, numeric and laboratory studies the reliability of the components and structures that utilize the materials in question prior to them leaving the design stage. The motivation behind these studies is to:

(a) be in a position to supply to a prospective customer, definitive instructions as to safety, inspection intervals, spare part availability, life cycle costs, etc., and

(b) be confident in these estimates based on accelerated and small scale laboratory testing and numerical algorithms.

This has been the expressed aim of an ongoing research program being currently undertaken in the Micromechanics of Solids Laboratory of McGill University. To date, this program has met with varied success but still requires further study prior to it being implemented with confidence on a large scale. One of the most promising outcomes of this research endeavour has been the Provan reliability law [1] whose derivation was based entirely on modelling the fatigue crack initiation and propagation processes by a statistical interference technique and by a Markov linear birth stochastic process, respectively. This reliability law in its failure density form is expressed as:

\[
p_N^{(i)} = \frac{\mu a_0}{2\pi V_{ac}} \exp \left\{ \lambda i - \frac{\left( \mu a_0 \exp(\lambda i) - \mu a_f \right)^2}{2V_{ac}} \right\}, \tag{1}
\]
with its associated cumulative expression being:

\[
P_{N_p}^{(j)} = \int_{-\infty}^{j} P_{N_p}^{(i)}(i) di = \frac{1}{2} \left\{ \text{erf} \left( \frac{\mu_{a_0} e^{\lambda i} - \mu_{af}}{\sqrt{2}V_{ac}} \right) + \text{erf} \left( \frac{\mu_{af}}{\sqrt{2}V_{ac}} \right) \right\}, \tag{2}
\]

where:

\[
\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-z^2} dz,
\]

\[
\text{V}_{ac} = \text{variance of crack penetration},
\]

\[
\mu_{a_0} = \text{mean of crack initiation depth},
\]

\[
\lambda = \text{growth rate transition intensity and}
\]

\[
\mu_{af} = \text{mean of critical crack depth}.
\]

Strictly speaking the reliability law is simply \( R_{j} = 1 - P_{N_p}^{(j)} \), but in keeping with common usage the expressions (1) and (2) are termed the reliability functions. The skewed to the left characteristic shape of the cumulative expression, for example, is illustrated by the solid line in Figure 1. Subsequent to the derivation of this reliability law, an extensive experimental program was entered into in order to check on the validity of these expressions \([2,3]\). They were performed on oxygen free high conductivity (O.F.H.C.) copper within the guidelines specified by the ASTM standard E466 \([4]\), under strain control at a total strain amplitude of \( \Delta \varepsilon / 2 = 0.003 \) and at a frequency of 0.5 Hz. The data collected in an increasing life cycle format are indicated by the dots in Figure 1. By curve fitting the Provan reliability law to the experimental points, the empirical value of the fundamental parameter known as the transition intensity was deduced as:

\[
\lambda_{\text{emp}} = 0.20 \times 10^{-3}/\text{cycle}; \quad \frac{\Delta \varepsilon}{2} = 0.003. \tag{4}
\]

As is observed, the agreement between the empirical and test results is reasonably good thereby confirming the validity of the approach being taken by this research investigation. However, as noted above, the whole idea of this research program is, that on the basis of a few accelerated laboratory tests and numerical algorithms, to be in a position to infer the reliability of both the material and, hence, the components manufactured from this metal. Two methods have previously been suggested \([5,6]\); one theoretical and the other experimental. The theoretical estimation procedure was shown in both \([2]\) and \([5]\) to seriously underestimate the number of cycles involved in initiating a fatigue crack and, as a result, the