IMAGING AND PROPAGATION OF BEAMS IN METALLIC OR DIELECTRIC WAVEGUIDES

Edward F. Kuester and David C. Chang
Department of Electrical and Computer Engineering
University of Colorado
Boulder, Colorado 80309 USA

ABSTRACT

Multimode dielectric or metallic waveguides have the property that a narrow beam launched into the waveguide does not randomly disperse, but forms repetitive cohesive patterns at certain points along the axis of the guide. This imaging property is the result of constructive interference between modes, and is mathematically related to the hybrid ray-mode formulations of Felsen and his colleagues. This paper will review the origins and applications of the imaging properties of uniformly filled metallic or dielectric waveguides.

1. INTRODUCTION

Multimode waveguides occur frequently in optical communications, underwater acoustics and other areas. Often a source of small extent (compared to the transverse dimensions of the waveguide) provides the excitation for the guide, resulting in a large number of modes being produced with significant amplitude. Field computation in such a case may be done by evaluating the amplitude of each mode, and summing all the modes together. In many applications, hundreds or even thousands of modes may be involved, and a large degree of cancellation of terms in the mode sum may occur. This can result in the accumulation of roundoff error when computer calculation is done, especially for large propagation distances.

The work of Felsen and his colleagues (1) on hybrid ray-mode expansions suggests that a more efficient representation in a ray expansion may be available, or even a yet more efficient combination of ray and mode terms. So long as a large number of mode terms can
be traded for a relatively small number of ray terms, this method provides significant improvement, but for extremely large values of the axial distance \( z \) (on the order of \( a^2/\lambda \), where \( a \) is the waveguide width and \( \lambda \) the wavelength) the number of rays required becomes comparable to the number of modes, and no improvement is obtained. The Gaussian beam excitation is handled within this framework by the use of complex source points (2), but this incurs additional difficulties when the beam is paraxial with the waveguide axis, which behaves almost as a focal-type caustic for the rays at large propagation distances.

A third approach to the problem tackles the partial differential equation by direct numerical means—using finite element or fast Fourier transform techniques (3). Once again, behavior at large distances is difficult to treat due to uncertain error accumulation in the process.

This paper summarizes an extension to the hybrid techniques which is based on imaging properties of waveguides: at certain distances from the source plane, a replica (or a specific combination or transform of such replicas) of the source will be produced in the paraxial approximation. Knowing the field at one of these imaging planes, the transform of the corresponding mode sum into a ray series can be used to calculate the field at locations sufficiently close to these planes. Since these planes are located at reasonably close intervals along the axis of the guide, it becomes possible to compute the field very accurately at nearly any point of the guide without any of the problems cited above.

2. FOURIER AND FRESNEL IMAGES IN WAVEGUIDES

2.1 Equivalence Between Metallic and Dielectric Guides

We limit our considerations to hollow (uniformly filled) metallic waveguides and step-index dielectric waveguides of various cross-sections. There is in fact a very general but simple equivalence between these two types of waveguide under the conditions we deal with here. When a large number of modes is present on a dielectric waveguide of any cross-sectional shape, the propagation constants and transverse field distributions of these modes are, to a good approximation, given by the propagation constants and longitudinal electric fields of the TM modes of a certain related metallic guide. The cross-section of the metallic guide is obtained from that of the dielectric guide by extending the core boundary normally outwards at each point by the distance \( k_0^{-1}(n_1^2 - n_2^2)^{1/2} \), where \( k_0 = \omega(\mu_0\varepsilon_0)^{1/2} \) is the free-space wavenumber,