THE SINGULARITY EXPANSION
REPRESENTATION OF SURFACE CURRENT
ON A PERFECTLY CONDUCTING SCATTERER

L. Wilson Pearson
Department of Electrical Engineering
University of Mississippi
University, MS 38677

1. INTRODUCTION

The term "singularity expansion" was coined by Baum [1] to apply to the characterization of the scattering phenomenon in terms of the complex natural resonances of the scattering object. The use of the concept of complex natural resonance appears in the literature from time to time previous to Baum's 1971 work (e.g. Schelkunoff [2]), but it is Baum's work that motivated a substantial effort during the 1970's to put the concept on a sounder mathematical footing and to apply it in a variety of applications and to a variety of geometries.

The singularity expansion characterizes the scattering phenomenon in terms of fundamental quantities intrinsic to a given scattering object—quantities in terms of which one may expand the surface current density induced on the object by a given incident field. Thence in turn one can express the scattered fields in terms of this surface current density. The quantities entering into the expansion are the complex natural resonances (or "poles") and corresponding natural current modes. These quantities depend on the global features of the scattering object, so that in the framework of alternative representations, they are well suited to the description of the scattering phenomenon at frequencies ranging over the first few resonant frequencies of the object. Equivalently, in the transient regime, they are well suited to computation of the induced current density at moderate and late times, but are ill-suited to early-time computation at least when time variations in the excitation waveform occur on time scales that are small compared with transit time across the object.
Mathematicians have given some attention to a rigorous theory of the singularity expansion for the scalar (acoustic) scattering case (e.g. see [3] and the bibliography therein), but the larger part of what has proceeded has appeared in the engineering science literature. As a consequence rigorous support is lacking in defining a number of features of the expansion and is supplanted by ad hoc arguments and at times only by numerical observation. In the present exposition we take the pragmatic point of view of the engineering scientist, calling on results which are felt to be trustworthy though at times non-rigorous. Where sound mathematical results are available, they are pointed out, however.

2. THE FORMAL SINGULARITY EXPANSION AND RAMIFICATIONS

2.1 The Singularity Expansion in the Frequency and Time Domains

We are concerned with the surface current density induced on a scattering object by an incident electromagnetic field as indicated generically in Figure 1. For present purposes we view the current on the scatter as being dictated by the electric field integral equation, though other formulations defining it are equally applicable. Defining the Laplace transform of the field quantities as

\[ E(r,s) = \int e(r,t) e^{-st} dt, \tag{1} \]

Figure 1. A scattering object under time-dependent excitation. The point \( r_0 \) is the "space-time origin" through which the wavefront passes at time \( t = 0 \).