MATHEMATICAL DIALECTICS, SCIENTIFIC LOGIC AND THE PSYCHOANALYSIS OF THINKING

As an experimental scientist, my interest in dialectics lies in its use as a logic of science and I chose this particular aspect of it in order to include a discussion of the work of Kosok and Gauthier at the Boston Symposium. At that time, I proposed to utilize dialectics as an intuitive model from which to construct mathematical structures applicable to evolutionary theory and to the logic of real thinking. I have not changed the article for publication: its message, I feel, is still valid and the task, unaccomplished. But, to accurately reflect my present viewpoint, I have added a brief discussion of dialectics as one of the natural patterns of thinking. The naturalistic observation of dialectical thinking may contribute to our understanding of philosophical dialectics; conversely, it suggests a possible contribution of dialectics to psychological science.

The establishment of the groundwork for the development of mathematical dialectics is the great value of the recent work of M. Kosok, Y. Gauthier, F. G. Asenjo, G. Günther, and a few others. Such an attempt transcends the mere formalization of the Hegelian system because mathematical dialectics as a logic of science must also be rooted in mathematics and in contemporary science. Therefore, it implies an actual revision and critique of Hegelian dialectics.

Kant proposed that discipline achieves the status of science only with the use of mathematical methods. Mathematization thus liberates science from empty speculation, dogmaticism and blind empiricism. In spite of its limitations, mathematical logic based on non-dialectic Aristotelian logic is used in computation, in mathematical research and as a logic of science. Such is not the case with dialectics that has not been so formalized. Thus, dialectics must become mathematical dialectics.

On the other hand, symbolic logic needs to incorporate dialectics since, as Hegel indicated in his famous dictum, rationality is the adaptation of thought to reality. Natural, historical, and psychological processes, as described by modern science, do not satisfy the postulates of classical non-dialectic ontology; this is in part described by what Kosok calls the 'non-linearity of science'. More generally, since the mathematical model of natural processes has required the development of many different mathematical structures,
mathematical logic cannot be limited to the logical interpretation of lattice theory as a calculus of propositions. Group theory, topology, etc., can also be given a logical interpretation. Such extended mathematical logic is equivalent to the formalization of some aspects of dialectic logic. In fact, dialectics is defined as the science of the most general laws of motion of matter, history, and thought (logic); mathematics is the formal science that constructs abstract models for natural processes including the most general laws of motion. Thus mathematics is mathematical dialectics if given an ontological and logical interpretation.

WHAT ARE THE ELEMENTS OF A MATHEMATICAL DIALECTICS?

In *The Science of Logic*, Hegel rejects the concept that logic makes abstraction of the content of thought. Thus, the elements of a mathematical dialectics, concepts or propositions, must be taken simultaneously in extension and in intension. Set theory can be used to develop a logic of the connotation of concepts in which simple attributes can be taken as elements and compound attributes as sets. The lattice of attributes is not isomorphic to that of classes (in contradiction to Boole's first law). For instance, there is an infinite number of attributes corresponding to the extensional empty set. Moreover, as the definition of a given class is more sharply defined, the definition of its connotation becomes less well-defined and vice versa. For instance, the well-defined attributes 'liquid' and 'solid', 'acid' and 'base', do not correspond to well-defined classes; a given substance can act as an acid if and only if there are other conditions under which it will act as a base. Extension and connotation are related by an uncertainty principle equivalent to the relation between position and momentum in Quantum Mechanics. Thus, a concept taken both in intension and extension satisfies Kosok's requirement that an element of a dialectic calculus cannot be well defined.

Let us then define a concept as an ordered pair \(<a, \bar{a}>\) the first element of which is a connotation (set of attributes) \(a\) and the second member in the corresponding class \(\bar{a}\) of objects. We now say that concept \(<a, \bar{a}>\) implies \(<b, \bar{b}>\) if and only if \(a < b\) and \(\bar{a} < \bar{b}\). This double implication solves many of the paradoxes of implication that plague classic Boolean logic and as a consequence, Hempel's theory of confirmation.

One needs to consider the connotation of concepts to formalize the Hegelian categories because all such categories are co-extensively universal. The interpretation of concepts in connotation allows for and requires the logical interpretation of more powerful mathematical structures than set