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HOW TO EXPLORE THE HISTORY OF ANCIENT MATHEMATICS?

I started investigations in the last two decades, which shed — I hope — some new light upon several problems of the history, as well as of the philosophy, of mathematics. Now I shall try to characterize here not so much the results of this research, as some of the problems dealt with and the methods I followed in these investigations.

Let me begin with a well-known classification, which led me to one of my most important problems. The different branches of scientific inquiry may be divided into two major groups: the empirical and the non-empirical sciences. The former seek to explore, to describe, to explain, and to predict the occurrences in the world we live in. Their statements are checked against the facts of our experience, and they are acceptable only if they are supported properly by empirical evidence. The empirical sciences are generally divided into the natural and the social sciences. (The behavioral sciences are, as I think, mainly social sciences, even if they do not exclude some very important influences on the part of the natural sciences. But this is not my problem now.) In any case, the dependence on empirical evidence distinguishes the empirical sciences from the non-empirical disciplines: logic and pure mathematics. For the propositions of logic and the mathematical theorems are proven without essential reference to empirical findings. The same division can, of course, be formulated also by saying that the method of all empirical sciences is induction, whereas both non-empirical disciplines, logic and mathematics, are deductive. Now, so far as mathematics is concerned, the last statement can be completed by a simple observation, particularly stressed in recent times by the mathematician Pólya, who reminded us that

mathematics has two faces; it is the rigorous science of Euclid but it is also something else. Mathematics presented in the Euclidean way appears as a systematic, deductive science; but mathematics in the making appears as an empirical, experimental, inductive science.

Indeed, most mathematical propositions were initially conjectures, mere guesswork on the basis of some empirical experience, and they became genuine theorems only later on, when mathematicians eventually succeeded in proving them, i.e., deducing them from some principles of mathematics.

The question which now raises itself is a double one: a philosophical and a historical one. (1) Philosophers have to answer the question: why is it that we appreciate the *deductive certainty*, that is, the proverbial certainty of proved mathematical propositions, more than the so-called *inductive uncertainty of the empirical sciences*? Speaking about this *uncertainty* I mean:

No matter how extensively an empirical hypothesis has been tested, and no matter how well it has been borne out by the test findings, it may yet fail in cases that have not been examined. Never does any empirical evidence suffice to verify a hypothesis, to establish its truth with *deductive certainty*; it can only lend the hypothesis more or less strong inductive support (Hempel).

No doubt, it is owing to our appreciation of ‘deductive certainty’ that mathematicians always strive to transform their conjectures, the mere guesswork, into genuine proved theorems. But I wonder whether the high esteem, in which we hold the so-called ‘deductive certainty’ is not exaggerated. So far as I know, there is not yet any satisfactory answer to the question, why mathematicians have to prove their statements, even in the case when these are obvious without any proof. As Pólya wrote:

We may say, with a little exaggeration, that humanity learned this idea — namely the idea of mathematical proof — from one man and one book: from Euclid and his *Elements*.

And so our second, the historical question: how did it come about that mathematics was transformed into a systematic, deductive science? For it was originally, in the ancient, pre-Hellenic cultures of Egypt and Babylon, only a highly developed practical, *empirical knowledge*. Indeed, it is generally accepted today that the Babylonians were able to find quite good approximations to the solutions of relatively complicated mathematical problems about a thousand years before the beginnings of Greek mathematics. Nevertheless, there is absolutely no evidence to suggest that the Babylonians (not to mention the Egyptians) ever tried to deduce mathematical theorems rigorously from first principles. It remains an open question, whether such fundamental scientific concepts as ‘theorem’, ‘proof’, ‘deduction’, ‘definition’, ‘postulate’, ‘axiom’, etc., were known to Egyptian and Babylonian mathematicians at all. It is not even certain that the Babylonians knew how to formulate general theorems. Mathematics prior to the ancient Greek civilization was nothing more than a useful, even sometimes an ingenious collection of practical rules, prescriptions as to how some particular mathematical tasks are to be carried out. The fundamental change in the history of mathematics, the