HEAT, COLD AND GEOMETRY

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0. INTRODUCTION

Classical and relativistic mechanics can be formulated in terms of symplectic geometry; this formulation leads to a rigorous statement of the principles of statistical mechanics and of thermodynamics.

This analogy also brings to light however certain fundamental difficulties which remain hidden in the traditional approach through some ambiguities.

The "first principle" of thermodynamics can be formulated so as to avoid this ambiguity provided one accepts a detour through the principle of general relativity and the Einstein equations for gravitation.

The mathematical tools used are the theory of symplectic moments, certain cohomological formulae and the concept of distribution-tensor.

As the "second principle" we shall merely show how it is possible, by accepting a particular geometry status for temperature and entropy, to construct a relativistic model of a dissipative continuous medium. This model has the following properties :

a. it is predictive;

M. Cahen et al. (eds.), Differential Geometry and Mathematical Physics, 37–68.

b. all fits solutions satisfy both principles of thermodynamics and admit a detailed balance (energy-impulse, momentum).

c. it contains in particular all equilibrium situations of statistical mechanics, and also the relativistic theory of elasticity.

d. Finally its non-relativistic limit allows one to identify the usual thermodynamic variables and in particular it contains the theory of elasticity, the mechanics of perfect fluids, the theory of heat conduction (Fourier) and the theory of viscosity (Navier).

Nevertheless it is a schematic model which does not take into account phenomena such as capillarity, plasticity, electromagnetic effects, etc.

I. SYMPLECTIC FORMULATION OF DYNAMICS

Consider first of all an elementary dynamical system : a newtonian point mass of mass m, position $\vec{r}$, velocity $\vec{v}$, in a force field $(\vec{r},t) \to \vec{F}$ (1); the triplet $y = (\vec{v},\vec{r},t)$ makes up an initial condition for a *motion* x; y travels through a manifold v, (évolution space); if one puts

$$\sigma_V(dy)(\delta y) = \langle m\, d\vec{v} - \vec{F}\, dt, \delta\vec{r} - \vec{v}\, \delta t\rangle - \langle m\, \delta\vec{v} - \vec{F}\delta t, d\vec{r} - \vec{v}\, dt\rangle \tag{1.1.}$$

d and δ being two arbitrary variations, the brackets $\langle,\rangle$ representing scalar product in $\mathbb{R}^3$, one defines on V, a 2-form σ_V of rank 6; the equations of motion become $dy \in \ker(\sigma_V)$ (2) ; if $\vec{F}$ is the gradient of some potential, σ_V is a *closed* form (its exterior derivative vanishes); σ_V is thus an absolute invariant integral of the equations of motion, discovered by E. Cartan, but in fact already described explicitly by Lagrange.

The set U of all possible motions has a structure of symplectic manifold (of dimension 6), provided with the closed and reversible 2-form σ_V, whose reciprocal image by the submersion $y \to x$ coincides with σ_V (fig. 1).

Such a scheme can be extended to general dynamical systems