THE MULTIPRODUCT LOT SCHEDULING PROBLEM

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ABSTRACT

An NP-hard problem of considerable practical interest is the multi-product lot scheduling problem. In its simplest form there are $P$ products to be scheduled on a single machine over a finite interval $(0, T)$. Associated with each product $i$ is a demand schedule $D_t$, a per unit time holding cost $h_t$, and a changeover cost vector $c_{ji}$, which is the cost of starting production on $i$ if the machine previously produced product $j$. In practical problems one might wish to treat the $D_t$ as random variables, although this feature is typically disregarded by solution procedures. Example situations might be a television manufacturer who produces several different styles and sizes of televisions on a single line or a chemical processor who produces several different chemicals in batches on a single expensive machine. We briefly summarize previous approaches to this problem starting with the work of Manne, Dzielinski, Gomory, Lasdon and Terjung and then analyze LP-like approximations to this model and provide bounds on the closeness of the LP solution to the exact IP solution as the problem size gets large.
INTRODUCTION

The lot scheduling problem is a generalization of the one machine sequencing problem. One wishes to determine which products should be produced when on a single machine so as to meet a specified demand schedule at minimum cost. The relevant costs are typically setup or changeover costs and inventory related costs.

Practical variations of the problem arise in the scheduling of appliances (e.g., TV's) on assembly lines or batches of different chemicals through an expensive chemical processor, or tires in a tire factory.

Analysis of lot scheduling problems have historically proceeded along one of two lines: a) the constant demand, constant costs, continuous time case and b) the dynamic demand and cost, discrete time case. Our analysis will be devoted mainly toward the dynamic demand, discrete time case. Elmagrerahby (1978) presents a recent survey of the continuous time case.

We will start by considering single product dynamic lot scheduling problems. Happily, it appears that single product lot scheduling algorithms can be used as subroutines in multiproduct lot scheduling algorithms. Unhappily, many of the interesting single product lot scheduling problems are already NP-hard without introducing the complication of additional products. It is the multiproduct problem which is of real practical interest.

DEFINITION OF THE SINGLE PRODUCT LOTSIZING PROBLEM

A single product dynamic lotsizing problem is described by the following set of parameters:

- $T =$ number of periods; the periods are indexed with $t = 1, 2, ..., T$,
- $E_t =$ maximum allowed inventory at the end of period $t$,
- $A_t =$ production capacity in period $t$,
- $S_t =$ setup cost incurred in period $t$ if there was production of the product in period $t$ but no production of the same product in period $t-1$,
- $R_t =$ an overhead or rental charge which is incurred, possibly in addition to $S_t$, if there is positive production of the product in period $t$,
- $C_t =$ production cost per unit produced in period $t$,
- $H_t =$ end of period holding cost per unit applied to positive end of period inventory,