Analysis of Thermally Stressed Variable Thickness Composite Discs—A CAD Technique

D. G. GORMAN

College of Engineering and Science,
National Institute for Higher Education, Limerick, Ireland

and

J. P. HUISSOON

Engineering Science School, Trinity College Dublin,
Dublin, Ireland

ABSTRACT

This chapter details a finite element technique for determining the state of in-plane stressing in a variable thickness composite disc when subjected to thermal loading and boundary restraints. By utilising two degrees of freedom annular finite elements of linearly varying thickness form, the numerical convergence rate is such that minimum computer storage is required, hence lending itself as the basis of a computer-aided design optimisation package. Laminated discs of either isotropic or polar orthotropic materials can be examined.

INTRODUCTION

Due to the wide range of engineering applications, the in-plane stress analysis of variable thickness discs has been the subject of much research this century. With the advent of high speed digital computers and advanced numerical techniques, Computer Aid Design (CAD) techniques with respect to this study have been extensively used in both industry and research establishments; for example, the technique developed by Seireg and Surana\(^1\) was used to compute the state of stressing over a turbine disc subjected to centrifugal loading only and consequently to establish an
optimum disc shape (axial thickness profile) which would ensure that the combination of the stress at any point of the disc would not exceed the yield condition. In the course of research into the free transverse vibration of thermally stressed discs, Gorman\textsuperscript{2} and Kennedy and Gorman\textsuperscript{3} extended the above technique to include the additional effect of thermal loading. In both these studies, however, only 'solid', isotropic specimens were considered, by means of subdividing the structure into a series of uniform annular rings and applying the Lamé theory to each ring, ensuring equilibrium and compatibility between adjoining rings. Using this technique for isotropic discs, an excellent degree of accuracy was obtained and, more important from the point of view of being a CAD technique, convergence was found to be rapid.

In the general analysis of composite discs, however, and in particular the analyses of polar orthotropic laminated discs, utilisation of the Lamé expressions, which greatly contribute to the fast convergence in the isotropic case, is no longer viable. In such cases a more generalised approach is required and in order to maintain a high rate of convergence it is necessary to pursue the analysis with a series of elements which render a closer approximation between the modelled and the actual structures than in the isotropic case, where elements of uniform thickness were used.

The aim of this chapter is therefore to demonstrate the technique whereby the in-plane stress distribution can be computed for a variable thickness laminated disc composed of either isotropic or polar orthotropic materials when subjected to any proposed temperature distribution. Consequently, for any set of limiting parameters (i.e. maximum yield stress of the materials), an optimum disc configuration may be computed. Additionally, since it is now well established that thermally induced in-plane stressing can dramatically change the vibratory and stability characteristics of continuous systems\textsuperscript{4} - \textsuperscript{6} in order to predict these changes it is necessary to establish the form of the in-plane stressing for any specified temperature distribution acting over the surface of the system.

\textbf{ANALYSIS}

(a) \textbf{Model Structure}

As shown in Figs 1(a) and (b), a variable thickness laminated disc may be modelled by a series of annular finite elements of uniform and linearly varying thickness form. Furthermore, for the case considered the two 'outer' laminates (material A) may be combined to form one structure with a common node at each extremity.