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RENORMALIZABLE THEORIES WITH SYMMETRY BREAKING

ABSTRACT. The description of symmetry breaking proposed by K. Symanzik within the framework of renormalizable theories is generalized from the geometrical point of view. For an arbitrary compact Lie group, a soft breaking of arbitrary covariance, and an arbitrary field multiplet, the expected integrated Ward identity are shown to hold to all orders of renormalized perturbation theory provided the Lagrangian is suitably chosen. The corresponding local Ward identity which provides the Lagrangian version of current algebra through the coupling to an external classical Yang–Mills field, is then proved to hold up to the classical Adler–Bardeen anomaly whose general form is written down. The BPHZ renormalization scheme is used throughout in such a way that the algebraic structure analyzed in the present context may serve as an introduction to the study of fully quantized gauge theories.

1. INTRODUCTION

Besides the well-known relevance of broken symmetries to elementary particle physics, further investigations of renormalizable models exhibiting broken symmetries are amply justified by the present understanding of gauge and even supergauge theories. The present status of the subject is well represented by K. Symanzik’s 1970 Cargèse lectures where the fundamental phenomena are discussed [1]. Since much of the structure analyzed there can also be found in the study of gauge theories, it is of interest to complete Symanzik’s analysis from the algebraic point of view, both in considering an arbitrary compact Lie group as describing the symmetry to be broken, and in analyzing the perturbative analog of current algebra, namely the coupling with a classical Yang–Mills field, which, as is well-known, leads to the definition of the celebrated Schwinger–Bell–Jackiw–Adler–Bardeen anomaly [2]. In fact, although this program had been announced by K. Symanzik, in collaboration with one of us (R.S.) [1], it has not been carried out until now following the stimulation [3] provided by the advent of gauge theories. Also, it seems that recent progress in renormalization theory has allowed a more tractable general treatment than the techniques known in 1970 would have permitted. Most of the present analysis relies on general properties of the perturbative series which stem from locality.
and power counting, as summarized by the renormalized action principle of Lowenstein and Lam [4] whose detailed form is one of the highlights of the Bogoliubov–Parasiuk–Hepp–Zimmermann [5] renormalization scheme. Within this framework, explicit bases of local operators of given dimensions are constructed [6], together with the linear relations connecting operators with different dimensions (the so-called Zimmermann identities). To avoid inessential technical complications, we shall only treat cases in which no massless field is involved. Our analysis can, however, be extended without essential modifications to a wide class of models involving massless fields exploiting Lowenstein’s and Zimmermann’s extension of BPHZ renormalization scheme [7]. Whereas these elementary tools, which are best exploited by means of a repeated application of the implicit function theorem for formal power series [3], suffice to solve most of the algebraic problems at hand, the elimination of some possible anomalies is occasionally performed by looking more deeply into the behaviour of the theory under scaling transformations, which provides some new non-renormalization type statements similar to that which leads to the non-renormalization of the Adler–Bardeen anomaly [8].

This article is divided into two main parts: Section 2 is devoted to the proof of the integrated Ward identity which expresses symmetry breaking for an arbitrary compact Lie group, with an arbitrary dimension (< 4) and covariance. Section 3 is devoted to a discussion of ‘current algebra’ which, in the present framework amounts to the proof of a local Ward identity, in the presence of a classical Yang–Mills field, and leads to the definition of the Adler–Bardeen anomaly.

A number of appendices are devoted to the treatment of some technical questions, among which the elimination from the integrated Ward identities of algebraically allowed anomalies consistent with power counting, and details about the cohomology [9] of the gauge Lie algebra associated with the symmetry group (i.e. the Wess–Zumino [10] consistency conditions).

2. Global symmetries broken

2.1. The Classical Theory

The general situation is as follows: G is a compact Lie group, $\mathfrak{g}$ its Lie algebra: $\mathfrak{g} = \mathfrak{g} + \mathfrak{a}$, $\mathfrak{g}$ semi-simple, $\mathfrak{a}$ abelian. $\varphi$ is a field multiplet.