ABSTRACT. The physical significance of coupling parameters in quantum field theories is questioned. As a result of the renormalization such couplings depend on parameters defining the renormalization procedure. On the other hand, the variation of these couplings when a renormalization parameter, like the scale of the renormalization mass for instance varies, makes physical sense when comparing physical strengths at different energies. This renormalization group machinery provides the possibility to go backwards in time in a Universe sprung from a big-bang for different theories of grand unification. It is this travel in the past which is explored in the present paper.

1. Generalities on coupling parameters

In 1953, a fundamental paper [1] questioned the notion of coupling parameters, used to expand in power series physical quantities in quantum field theories. This was somewhat revolutionary at a time when the fine structure constant in QED in the limit of vanishing energy momentum (from $2P_{3/2} - 2P_{1/2}$ separation in deuterium or from the Thomson limit etc.) seemed to be the only possible definition and was moreover consistent with the classical definition of the electron charge via the correspondence principle.

In the summary, at the beginning of this 1953 paper, one can read indeed the following:

... A coupling parameter, however, can only be specified in terms of a chosen development of a function $S(xy; c_1, \ldots)$ of physical significance. However, the terms of the actual correspondence development (in terms of $e^2$) $S = S_2 + S_4 + \ldots$ have no physical meaning. Therefore the coefficient $e^2$ in $S_2$ has only a mathematical significance. It requires that the functions of $S_4, S_6, \ldots, S_n$ have all been specified. As this specification involves the $c_i$'s, we must expect that a group of infinitesimal operations $P_i = (\partial/\partial c_i)_{c=0}$ exists, satisfying

$$P_i S = h_i(\kappa, \mu, e) \frac{\partial}{\partial e} S(\kappa, \mu, e; 0, 0)$$

admitting thus a renormalization of $e$.

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* $\kappa$ stands for the electron mass.
** $c_i$ are parameters originating in finite renormalizations.
*** $S$, appearing on the left is of course the $S(\kappa, \mu, e; c_1, c_2, \ldots)$. 

The above equation is the general form of the renormalization group equations for QED. Its restriction to the trivial one-parameter group for overall scale transformations of the subtraction point $\mu$

$$\mu' = \mu \exp (-c) \quad \text{(1)}$$

is obvious. It gives the usual renormalization group equation for observable physical quantities

$$-\mu \frac{\partial}{\partial \mu} S_0(e, \lambda, \mu) = \beta \frac{\partial}{\partial e} S_0(e, \lambda, \mu) \quad \text{(2)}$$

(with $\beta = + h_e$ and $S_0(e, \lambda, \mu; c = 0)$ since

$$\left. \frac{\partial}{\partial c} S_0(e, \lambda, \mu \exp (-c)) \right|_{e, \mu, \mu\text{ fixed}} = -\mu \left. \frac{\partial}{\partial \mu} S_0(e, \lambda, \mu) \right|_{e, \mu, \mu\text{ fixed}}$$

Therefore, Equation (2) is, as announced, exactly equivalent to

$$\left. \frac{\partial}{\partial c} S(e, \lambda, \mu; c) \right|_{e, \mu, \mu\text{ fixed}} = h_e \left. \frac{\partial}{\partial e} S(e, \lambda, \mu; 0) \right|_{e, \mu, \mu\text{ fixed}} \quad \text{(4)}$$

the Equation (I) of the summary quoted above. Indeed, for the special subgroup considered (scale transformation of $\mu$)

$$S(e, \lambda, \mu; c) = S_0(e, \lambda, \mu \exp (-c)) \quad \text{(5)}$$

This kind of equation* has played various roles since then in several topics of physics. For the first time, one had an equation with a derivative with respect to a coupling parameter, allowing thus to relate different orders in the perturbation expansion amongst themselves. This feature has been explored and exploited to determine the asymptotic behaviour of all terms in perturbation theory. But the most exciting aspect is that it questions seriously the concept of coupling constant. Indeed, since physical quantities must satisfy relations like

$$S_0(e, \mu \exp (-c)) = S_0(e(c), \mu) \quad \text{(6)**}$$

*Equation (I) extracted from the summary of Ref. 1. For multiplicatively renormalized quantities (Green functions, 1PI amplitudes, etc.):

$$S(e, \lambda, \mu; c) = z^\lambda(c) S_0(e, \lambda, \mu \exp (-c))$$

there is an additional term in (I): $K(\partial z(c)/\partial c) \big|_{c=0}$ (the so-called anomalous dimensions $\gamma$); $z^\lambda(c)$ stands for a product of various powers of $z$'s with $z(0) = 1$.

** $\times$ has from now on been set equal to zero.