STATE SPACE MODELS FOR GAUSSIAN STOCHASTIC PROCESSES

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ABSTRACT: A comprehensive theory of stochastic realization for multivariate stationary Gaussian processes is presented. It is coordinate-free in nature, starting out with an abstract state space theory in Hilbert space, based on the concept of splitting subspace. These results are then carried over to the spectral domain and described in terms of Hardy functions. Each state space is uniquely characterized by its structural function, an inner function which contains all the systems theoretical characteristics of the corresponding realizations. Finally coordinates are introduced and concrete differential-equation-type representations are obtained. This paper is an abridged version of a forthcoming paper, which in turn summarizes and considerably extends results which have previously been presented in a series of preliminary conference papers.

1. INTRODUCTION

In recent years there has been a considerable interest in various versions of the so-called stochastic realization problem [1-31], which, loosely speaking, can be described as the problem of finding (a suitable class of) stochastic dynamical systems, called realizations, all having a given random process \{y(t); t \in T\} as its output. (Here T is the index set, which usually is the real line R or the set Z of integers.) In the past it has often been
assumed that $y$ is a stationary (or stationary increment) process with rational spectral density, thus insuring the existence of finite dimensional realizations.

The early papers on the subject [1-3] consider a deterministic version of the problem, the objective being to realize (in the deterministic sense [32,33]) the spectral factors of the given process which is defined up to second-order properties only. The probabilistic aspects of the stochastic realization problem were subsequently clarified in [8-10]. In all these papers the states of the realizations are represented in a fixed coordinate system to avoid trivial questions of uniqueness.

However, the most natural approach to the stochastic realization problem is coordinate free: Begin by constructing families $\{X_t; t \in T\}$ of state spaces which evolve in time in a Markovian manner. These state spaces should be as small as possible, but large enough to contain the essential information for determining the temporal evolution of the given process. Then, for each such family, concrete realizations can be obtained by introducing suitable bases in the state spaces. This line of study was initiated in [4-6], where a restricted version of the problem of this paper was studied, considering only state spaces contained in the closed span of the past (or, symmetrically, the future) of the given process. With such a state space approach we need not restrict the analysis to processes with rational spectral density, since the framework will also accommodate infinite dimensional state spaces.

During the last couple of years we have been developing a state space theory of stochastic realization which is now in a reasonably complete form. Part of our work has been reported in a series of preliminary conference papers [11-15]; a more complete account will appear in a forthcoming paper [16], which is now under preparation. Some results in the first phase of this work were obtained in cooperation with Ruckebusch [17], who parallelly developed his own geometric state space theory [20,21]. The present paper is an attempt to summarize the results presented in [16]. Due to page limitations, not all topics of [16] will be discussed. Also we have left out the proofs of the theorems, instead providing the reader with references for the proofs. For simplicity, only realizations of continuous-time stationary processes will be discussed, but it should be understood that our basic geometric theory holds also for stationary increment processes and discrete-time processes, and that the subsequent spectral theory can be appropriately modified to take care of these cases also.