CHAPTER 7

LANGUAGE AND INDUCTIVE LOGIC

In an earlier chapter it was seen that language and deductive logic were intimately related; that, in fact, every language determined a unique deductive logic. But what of inductive logic? We turn now to that question and the closely related problem of how to represent information believed only with a certain degree of probability.

7.1 CREDIBILITY WEIGHTS

The language automata considered up to this point have been two-valued; only the two credibility levels of belief and its absence have been distinguished. It has been assumed that a heard sentence is either received and accepted by the language automaton or not received by it at all, the input selector acting as an all-or-nothing screening device. Likewise output has been assumed to consist of yes-or-no signals to indicate whether a sentence is currently affirmable or not. These assumptions are restrictive and it is natural to generalize to the many-valued case.

Let us assume then that a set of numbers or other entities called *credibility weights* is specified as part of the language automaton. Intuitively a credibility weight signifies a degree of believability or assertability. Each informative input sentence is accompanied by a credibility weight which serves as a signal from the input selector to the language automaton instructing it as to how strongly the sentence is to be believed. Each output given in response to a test input is also a credibility weight, interpreted this time as an indication of the level of belief already accorded the sentence in the current state. The two-valued case is then the special case in which the credibility weight set contains just two weights interpreted as belief and nonbelief.

The most convenient assumption to make about the credibility weight set is that it contains a continuum of values, the interpretation being that the credibility levels range from absolute belief or certainty to absolute disbelief. Such a weight set is representable mathematically as the real line or an interval on the real line. This model, though not necessarily the most realistic psychologically, is interesting because one feels intuitively that degrees of belief are indeed graduated continuously like shades of grey.
A continuous spectrum of belief values overcomes certain behavior-analytic anomalies of the two-valued model. For example, the utterance of a sentence may be less likely when it is strongly disbelieved than when it is weakly disbelieved or when the speaker feels neutral about it, for some speakers may avoid lying more carefully than bluffing. Such output selection criteria would not be possible on the basis of mere belief–nonbelief signals from the language automaton. Similarly a preference for uttering very strongly believed sentences over those that are only fairly strongly believed could be explained only in terms of a sufficiently finely graduated credibility weight set.

In the generalized theory a language automaton becomes a weighted language automaton and a weighted language is an equivalence class of weighted language automata. The notions of a basic evidence statement, of stability, receptivity, intransigence, and so on are readily extended to weighted languages. Nonpathological languages are again stable and receptive.

A weighted language automaton has the form of a 5-tuple \( \langle S, W, Z, L, C \rangle \) where \( S \) is a sentence set, \( W \) is a simply ordered set containing at least two members (the 'credibility weights'), \( Z \) is the state set, \( L \) is a (partial) function \( L: Z \times S \times W \rightarrow Z \) and \( C \) (the 'credibility function') is a (partial) function \( C: Z \times S \rightarrow W \). Since the members of \( W \) are indirect observables their mathematical structure may be chosen arbitrarily; taking them to be numbers is convenient in that it makes the simple ordering self-evident. Such a 5-tuple is the 'abbreviated' form of a weighted language automaton; its precise definition in the standard automata-theoretic form of Definition 2.1 is left to the reader. When translated into the standard form the standard concept of equivalence (Definition 2.5) applies to yield the definition of a weighted language.

If \( S \) is a sentence set and \( W \) a credibility weight set, a weighted basic evidence statement in \( S \) and \( W \) is a metalanguage statement of form

\[
\text{There exists (doesn't exist) a } z \in Z \text{ such that } C(L(z, \bar{s}_{1}, \bar{w}_{1}), s_{1}) = w_{1} \text{ and } \ldots \text{ and } C(L(z, \bar{s}_{n}, \bar{w}_{n}), s_{n}) = w_{n}
\]

where \( 1 \leq n < \infty \) and for each \( i, 1 \leq i \leq n, s_{i} \in S, \bar{s}_{i} \) is a finite (possibly null) sequence of members of \( S \), \( w_{i} \in W \) and \( \bar{w}_{i} \) is a sequence of members of \( W \) of length equal to the length of \( \bar{s}_{i} \). The virtues of the unweighted basic evidence statements as specific behavioral properties coming as close could be hoped to providing a complete basis for language investigation carry over to the weighted basic evidence statements.