THEORY OF ATOMIC HYPERFINE STRUCTURE

Anatoly V. Andreev

Physics Department, M.V.Lomonosov Moscow State University, Moscow 119992 Russia E-mail andreev@sr1.phys.msu.su

Abstract The new equation for $\frac{1}{2}$ spin relativistic particle is proposed. In comparison with Dirac equation the proposed equation (i) is completely symmetric with respect to particle and its antiparticle, (ii) predicts the hyperfine structure of electron energy levels in Coulomb field of nucleus with zero magnetic moment, (iii) explains the Lamb shift and electron anomalous magnetic moment without secondary quantization either the matter or the radiation field. The analytically tractable solutions of the proposed equation are found for the problems of electron motion in the Coulomb field, in uniform magnetic field and in superposition of Coulomb and uniform magnetic fields.

1. Introduction

The hydrogen atom spectrum calculated with the help of Dirac equation is in good agreement with results of its experimental measurements. This is one the main evidence of the fundamental nature of the Dirac theory. Nevertheless the real energy-level diagram of hydrogen atom is still wealthy than this predicted by Dirac equation. In addition to fine structure the hydrogen spectrum includes the shift in fine structure (Lamb shift [1]), hyperfine structure of levels, anomalous value of electron magnetic moment [2], etc. At the present time all of these specific features of the hydrogen atom spectrum have the reasonable explanations. The hyperfine structure of levels is explained by interaction of electron spin with nucleus spin spin [2]. The Lamb shift [3] and anomalous magnetic moment [4] are explained by interaction of electron with vacuum fluctuations of electromagnetic field (radiation corrections) and their calculated values are in good agreement with the experimental data. It should be noted that a number of attempts were made to explain the Lamb shift and anomalous magnetic moment avoiding quantization of the field. Jaynes and co-workers have constructed nonquantized- field "neoclassical" theory [5] based on the old Schrodinger interpretation of quantum mechanics. Eberly and co-wokers [6] have developed the radiation reaction or source-field approach in quantum electrodynamics. Barut and co-workers [7] have developed the quantum electrodynamics based

on self-fields. The complex structure of spectrum of hydrogen atom, which is one of the simplest physical objects, makes evident the necessity for search of ways to revise the Dirac theory. The form of Hamiltonian of Dirac equation, including the first degree of electron momentum, gives additional arguments for revision of this equation. Indeed, the energy of free particle should not depend on the direction in which the particle moves.

It is well known that the Dirac equation can be produced in result of splitting of the second order differential equation into the two first order differential equations

$$\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \Delta + \frac{m_0^2 c^2}{\hbar^2} = \left[ \frac{1}{c} \frac{\partial}{\partial t} + \vec{\alpha} \cdot \vec{\nabla} + \frac{im_0 c}{\hbar} \beta \right] \left( \frac{1}{c} \frac{\partial}{\partial t} - \vec{\alpha} \cdot \vec{\nabla} - \frac{im_0 c}{\hbar} \beta \right),$$

where $\alpha$ and $\beta$ are Dirac matrices. One of these first order operators is then associated with the negatively charged particle and another with the positively charged. On the other hand it seems that this splitting could be associated with the splitting of two counter propagating waves that appear when we solve the second order differential equation. Indeed the Hamiltonian Hamiltonian of monodirectional propagating component should depend on the momentum but not on its magnitude. In such a case we shall not meet any problems in analysis of the free motion of particle, but the serious problems may arise when we apply this equation for the study of bound states of the particle in the external field.

Recently [8] we proposed the equation for action for spin $1/2$ relativistic particle interacting with the electromagnetic field. The proposed action leads to the second order differential equation for the bispinor electron wave function. The obtained equation is symmetric with respect to the negatively and positively charged particles. This symmetry reflects the equivalence of physical properties of particle and its antiparticle. The present paper consists of two main parts. In the first part we study the general properties of the proposed equation and then we discuss a number solutions of this equation that were obtained in analytically tractable form. The main distinctive features of the proposed equation can be easily understood from the comparison with the Dirac equation. One of the most important features that follows from this comparison is in the fact that the all solutions of the first order differential Dirac equation are at the same time the solutions of the proposed second order equation. But the second order equation have the additional solutions that are not the solutions of Dirac equation. There is a technique in quantum electrodynamics (see for example [9]) when we solve initially the second order differential equation and then we retain only those solutions that satisfy the first order Dirac equation. However there is a principle difference between these two equations when we study the interaction of particle with electromagnetic field. The proposed equation for action result not only in the second order differential equation for bispinor wave function of particle, but it generates also the new equations for charge and current density.