NISHIMORI POINT IN RANDOM-BOND ISING AND POTTTS MODELS IN 2D

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Abstract We study the universality class of the fixed points of the 2D random bond $q$-state Potts model by means of numerical transfer matrix methods. In particular, we determine the critical exponents associated with the fixed point on the Nishimori line. Precise measurements show that the universality class of this fixed point is inconsistent with percolation on Potts clusters for $q = 2$, corresponding to the Ising model, and $q = 3$.

Keywords: Spin glasses, Potts model, Nishimori line, conformal field theories

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1. Introduction

During the last decade, the study of disordered systems has attracted much interest. This is true in particular in two dimensions, where the possible types of critical behavior for the corresponding pure models can be classified using conformal field theory [1]. Recently, similar classification issues for disordered models have been addressed through the study of random matrix ensembles [14], but many fundamental questions remain open.

An important category of 2D disordered systems is given by models where the disorder couples to the local energy density (random Potts models). Here we shall study such models that interpolate between ferromagnetic random bond disorder, and a stronger $\pm J$ type disorder. Our main focus shall be on the cases with $q = 2$ (Ising) or $q = 3$ states.

2. Phase diagram

The Ising model on a square lattice is one of the most popular two-dimensional systems. It is specified by the energy of a spin configuration:

$$\mathcal{H}(\{S_i\}) = \sum_{\langle i,j \rangle} J_{ij} \delta_{S_i,S_j},$$  \hfill (1)

where the sum is over all bonds and the coupling constants $J_{ij}$ are bond dependent. Different distributions of disorder can be considered. The most common ones are the $J_{ij} = \pm 1$ and the Gaussian distribution of disorder. In this work we will study in particular the $J_{ij} = \pm 1$ Random-Bond Ising Model (RBIM) with the following probability distribution:

$$P(J_{ij}) = p\delta(J_{ij} - 1) + (1 - p)\delta(J_{ij} + 1).$$  \hfill (2)

The topology of the phase diagram of the RBIM depends crucially on the type of disorder one considers. An instructive example is provided by a disorder having only two possible values for the bonds with equal signs and probabilities. It is by now well established [3] that the only non-trivial fixed points are located at the extrema of the boundary of the ferromagnetic phase, corresponding to the pure Ising fixed point and a zero temperature fixed point which turns out to be in the percolation universality class.

When the distribution contains also bonds with different signs (like in (2)), the situation is more subtle. In particular, it is known since the work of McMillan [4] that there exists an unstable fixed point at finite temperature and finite value of disorder $p_c$ and another fixed point at zero temperature and a value of disorder $p \approx p_c$ (McMillan obtained these results with a Gaussian distribution of disorder). Thus for the RBIM, one expects three fixed points (see Fig. 1a): i) the fixed point corresponding to the case without disorder. Close to this point,