SUB-MILLIMETER EXTRA DIMENSIONS AND TEV SCALE QUANTUM GRAVITY

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Abstract. In this lecture we will briefly review some phenomenological, astrophysical and cosmological aspects of theories with large extra dimensions and low scale quantum gravity.

An extraordinary weakness of gravity in comparison with all the other known subatomic forces is a great mystery in fundamental physics. For decades, the standard paradigm has been that there is a giant "desert" in energy scales over seventeen orders of magnitude. This huge discrepancy in scales is the so called hierarchy problem. This desert stretches from energies of order 100 GeV, currently probed at existing particle physics accelerators, all the way up to energies of order $10^{19}$ GeV (or length scales of order $10^{-33}$ cm) where, according to this view, gravity should "catch up" and unify with the other interactions.

Recently this paradigm has been challenged in [1, 2]. It was shown that the scale of quantum gravity can be 16 order of magnitude smaller than what was expected, and thus be accessible at present and future high energy accelerators. This is accomplished by postulating the existence of large extra dimensions of sub-millimeter size.

Of course, the idea that our world may have extra space dimensions, in addition to the three "obvious" ones that we see, is not new. Until very recently, however, these extra dimensions were assumed to be curled up into tiny circles about $10^{-33}$ cm in size, killing any hope for experimentally detecting them.

This size estimate comes from assuming the size is determined by the value of Newton's gravitational constant $G_N \sim (10^{-33} \text{cm})^2$. This sets the length scale at which, according to Newton's law, the gravitational force
between elementary particles

$$F(r) \sim G_N \frac{m_1 m_2}{r^2}$$  \hspace{1cm} (1)

becomes strong. In our framework, the extra dimensions, instead of being \(10^{-33}\) cm, can be as large as a millimeter, and thus accessible not only at particle accelerators, but also in proposed table-top experiments measuring gravity at sub-millimeter distances.

To explain how this may be possible, let us ask for a moment what we know about the gravitational interaction experimentally. Due to its miniscule strength, we know surprisingly little: gravity has only been measured down to distances of about a millimeter. The old paradigm relies on assumption that the Newtonian gravitational (1) is not modified from centimeter distances (where it is measured already) all the way down to \(10^{-33}\) cm.

Our framework postulates that quantum gravity becomes strong at a scale \(M_{Pf} \sim \text{TeV}\) not much above the electroweak scale. The observed weakness of gravity on distances \(\geq 1\) mm is attributed in our scenario to the existence of new space dimensions of sub-millimeter size to which gravitational force lines can spread. There are several ways to see how the extra dimensions can "weaken" gravity at large distances. The simplest perhaps is the Gauss law. Imagine that there are \(N\) extra compact dimensions of size \(\sim R\). We shall assume that roughly all of them have an equal size. Precise shape of the compact manifold is not important for our purposes. Now consider two point-like test masses \(m_1\) and \(m_2\) in this Universe. Obviously, if the distance between particles \(r\) is smaller than the size of extra dimensions \(R\), these particles see the Universe being effectively \(4 + N\)-dimensional and the gravitational force between them will be governed by an \(4 + N\)-dimensional Gauss law:

$$F(r) \sim G_{Nf} \frac{m_1 m_2}{r^{2+N}}$$  \hspace{1cm} (2)

where \(G_{Nf} = M_{Pf}^{-(2+N)}\) is the fundamental Newtons constant, and we can define \(M_{Pf}\) as the fundamental Planck scale. Solution of the hierarchy problem then sets \(M_{Pf} \sim \text{TeV}\).

Now, at distances \(r \gg R\), gravitational flux can not spread in extra dimensions and we should recover an usual four-dimensional Newtons law (1). The relation between observed \(G_N\) and the fundamental \(G_{Nf}\) Newtonian constants is given by [1]

$$G_N \sim \frac{G_{Nf}}{V_N}$$  \hspace{1cm} (3)

where \(V_N \sim R^N\) is the volume of the transverse high dimensional space. For instance, in case of two large dimensions, \(V_2 \sim (1\text{mm})^2\) up to factors