A USEFUL FOUR-VALUED LOGIC BY NUEL D. BELNAP, JR.

Nuel D. Belnap, Jr, and his close colleague of many years, the late Alan Ross Anderson, are the founding fathers of what has been referred to at least once in print as the 'Pittsburgh school' of logic. Working from earlier ideas of Wilhelm Ackermann, they have developed their systems **E** of Entailment and **R** of Relevant Implication, both of which require that for an implication 'if A then B' (in symbols $A \to B$) to hold, there must be some connection of meaning (some 'relevance') between A and B (**E** requires that the implication be necessary as well – thus **E** is a modal as well as a relevant logic). For sentential logic such a connection of meaning expresses itself formally in the famous Anderson-Belnap Variable Sharing Criterion, which requires that if $A \to B$ is a theorem, then A and B must share some sentential variable. This criterion is satisfied by both the systems **E** and **R**, but not by the more familiar systems of logic, eg., classical logic, modal logic, intuitionistic logic, and Łukasiewicz many-valued logic. All of the familiar systems buy the principles that a theorem is implied by any sentence whatsoever, and that the negation of a theorem implies any sentence whatsoever, whether or not there is any sharing of meaning.

The last principle is the more important in the context of introducing Belnap's present paper, wherein he suggests "a useful four-valued logic" to be used by computers performing deductions on quite possibly inconsistent data bases. Thus, in particular, Belnap does not want this logic to license that any conclusion whatsoever can be drawn from a contradiction. To use our own example, we would not want the F. B. I.'s computer to draw the conclusion that we are traitorous spies merely from the fact that different informants gave slightly different versions of our children's ages.

Belnap goes out of his way in the present paper not to assume on the part of the reader any prior knowledge of, or sympathy for, the grand enterprise of relevant logic (he does at least hint that there is such a grand enterprise, with his remark at the end about the 'One True Logic'). We refer the reader who would like to know more about relevant logic to Anderson and Belnap's *Entailment* (vol. 1), Princeton University Press, 1975.

J. M. Dunn and G. Epstein (eds.), Modern Uses of Multiple-Valued Logic 5–37.

Belnap usefully divides his paper into three parts, after the introduction which motivates and delimits the nature of the computer application which he has in mind. (Incidentally, Belnap notes that the introduction and Part I are spelled out in more detail in another paper (Belnap, 1976).) Part I is concerned with the case where the computer accepts information regarding the truth or falsity of only atomic statements, e.g., the Pirates won the 1971 World Series. Part II deals with the case of truth-functional compounds of atomic statements, e.g., the Pirates or the Orioles won. And Part III investigates certain non-truth functional implications construed as rules for improving the computer's data base, e.g., if the Pirates won, then the Orioles did not.

It would be out of place in a preface such as this to go deeply into the mathematical structures that Belnap explores in modelling his ideas. They get progressively more complex from Part I to Part III. But we shall say just a little about the basic mathematical notions of Part I. The starting point is that a computer might be in any one of four different epistemic states with respect to some given atomic statement, i.e., the computer could have simply been told that the statement is true, it could simply have been told that the statement is false, it could have been told neither, *and it could have both been told that the statement is true and also told that the statement is false.* The first two epistemic states are utterly familiar, and even the third should have a fairly homey feel to it because of Łukasiewicz 3-valued logic, but we felt the need for italics in describing the fourth state because of its novelty. (The novelty, we stress, lies entirely at the 'theoretical' level – at a 'practical' level the phenomenon of inconsistent information is, regrettably, well-known.)

Belnap mathematically reifies his fourfold division as four 'values': ***T***, ***F***, ***None***, ***Both***. These can be identified with the four subsets of the usual set of two truth values {truth, falsity}. Belnap's values are intended as epistemic, rather than ontic, in character. It is not that sentences are sometimes *really* neither true nor false, or both true and false; it is just that sometimes the computer is *told* such. It may be useful to recall for the point of the contrast that Łukasiewicz did intend an ontic interpretation of his third value '***None***' (with respect to certain statements about the future), although epistemic interpretations of Łukasiewicz's third value have been popular among many later workers in multiple-valued logic.

Belnap observes that a lattice structure may be put upon the four values in two different (albeit isomorphic) interesting ways. The first way produces an 'approximation lattice' (in the sense of Scott) **A4**, with ***None*** at the bottom, ***Both*** at the top, and ***T*** and ***F*** as incomparable points in between. The second produces a 'logical lattice' **L4**, with ***F***