CHAPTER XV

CLASSES AS ABSTRACT ENTITIES
AND THE RUSSELL PARADOX

I

The implications of the theory of abstract entities which was developed
in the paper with that title\(^1\) for some classical problems pertaining to
classes are interesting and, I believe, important. To show this I shall take
my point of departure from the thesis, advanced in an earlier paper,\(^2\) that
the form

\[ x \in k \]

construed as the logistical counterpart of

\[ x \text{ is a } k \text{ (e.g., } x \text{ is a man) } \]

must not be confused with the same form construed as the logistical
counterpart of

\[ x \text{ is a member of } k\text{-kind} \]

where "\(k\)-kind" is given a technical use as a referring expression which
refers to a class, and in which it is equivalent to "the class of \(ks\)."
According to our analysis

\[ x \text{ is a } k \]

stands to

\[ x \text{ is a member of } k\text{-kind} \]

as

\[ f(x) \]

to

\[ f\text{-ness is true of } x \]

or, as it is often put,

\[ x \text{ exemplifies } f\text{-ness}. \]
I shall use \( E_1 \) as the logistical counterpart of "is a" and \( E_2 \) as the counterpart of "is a member of." The latter, then, is a relation word closely related in sense to "exemplifies," whereas \( E_1 \) is not a relation word, but is rather a syncategorematic expression which is, to use an Irish Bull, a part of the common noun which follows it. Thus, to use a familiar mode of representation,

\[
\text{Tom is a man}
\]
does not have the form

\[
(x) \ R(y)
\]
but rather

\[
f(x)
\]
and would be appropriately symbolized as

\[
E_1 k(x) \quad \text{thus, is-a-man (Tom)}
\]

instead of the more familiar, but less perspicuous,

\[
x \ E_1 k.
\]

The first thesis I wish to advance can be summed up as follows. The form

\[ -E_1 - \]
requires a singular term in the left hand space and a common noun in the right hand space. On the other hand, the form

\[ -E_2 - \]
requires a singular term in each space.\(^3\) In other words, the forms

\[
k \ E_1 k \\
k \ E_2 k \\
k \ E_2 \ \text{k-kind}
\]
are all ill-conceived, where \( k \) is intended to function as a common noun, and \( E_1 \) and \( E_2 \) to have the meanings specified above. There are, however, well-formed expressions in the neighborhood which can, and will, be confused with these if one overlooks the significant differ-