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IS LOGIC EMPIRICAL?

I want to begin by considering a case in which 'necessary' truths (or rather 'truths', turned out to be falsehoods: the case of Euclidean geometry. I then want to raise the question: could some of the 'necessary truths' of logic ever turn out to be false *for empirical reasons*? I shall argue that the answer to this question is in the affirmative, and that logic is, in a certain sense, a natural science.

I. THE OVERTHROW OF EUCLIDEAN GEOMETRY

Consider the following assertion (see Figure 1): two straight lines AB and CD are alleged to come in from 'left infinity' in such a way that, to the left of EF, their distance apart is constant (or, at any rate 'constant on the average'), while after crossing EF they begin to converge – i.e. their distance apart diminishes – without its being the case that they bend at E or F (i.e., they are really straight, not just 'piecewise straight').

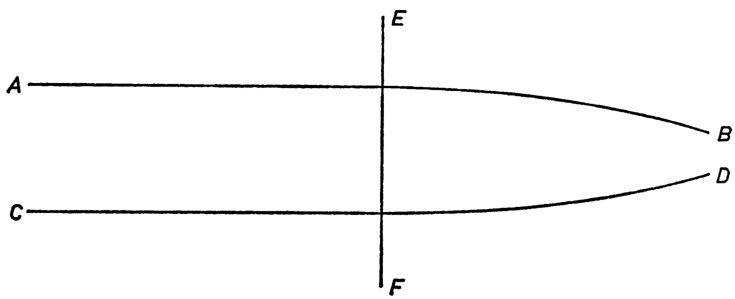

Fig. 1.

Is it not 'intuitively clear' that this is a contradiction? If one did not know anything about non-Euclidean geometry, relativity, etc., would the intuitive evidence that this is a contradiction, an impossibility, a complete absurdity, etc., be any less than the intuitive evidence that no surface can

be scarlet (all over) and bright green at the same time? Or that no bachelor is married? Is the intuitive 'feeling' of contradiction really different in the following three cases?

(1) Someone claims that *AB* and *CD* are both *straight lines*, notwithstanding their anomalous behavior.

(2) Someone claims that he has a sheet of paper which is scarlet (all over, on both sides) and green (all over, on both sides) at once.

(3) Someone claims that some men are married (legally, at the present time) and nonetheless still *bachelors*.

It seems to me that it is not. Of course, (1) does not involve a 'contradiction' in the technical sense of formal logic (e.g, 'p · – p'); but then neither does (2), nor, unless we stipulate the definition 'Bachelor = man who has never been married', does (3). But then 'contradiction' is often employed in a wide sense, in which not all contradictions are of the form 'p · – p', or reducible to that form by formal logic alone.

The important thing, for present purposes, is this: according to General Relativity theory, the claim mentioned in (1) is *possible*, not *impossible*. *AB* and *CD* could both be *straight paths* in the strict sense: that is, for no P, P' on *AB* (or on *CD*) is there a shorter way to travel from P to P' than by sticking to the path *AB* (respectively, *CD*). If we are willing to take 'shortest distance between two points' as the defining property of straight lines, then *AB* and *CD* could both be straight lines (in technical language: *geodesics*), notwithstanding their anomalous behavior.

To see this, assuming only the barest smattering of relativity: assume space is Euclidean 'in the large' – i.e., the average curvature of space is zero. (This is consistent with the General Theory of Relativity.) Then two geodesics could well come in from 'left infinity' a constant distance apart on the average ('on the average' mind you! – and I am speaking about straight lines!). Suppose these two geodesics – they might be the paths of two light rays approaching the sun on opposite sides[1] – enter the gravitational field of the sun as they cross *EF*. Then, according to GTR, the geodesics – not just the light, but the very geodesics, whether light is actually travelling along them or not – would behave as shown in Figure 1.

Conclusion: what was yesterday's 'evident' impossibility is today's possibility (and even *actuality* – things as 'bad' as this actually happen,