TRANSITION TO TURBULENCE

As already stressed in Chapter I, turbulence can only develop in rotational flows: it is due to the existence of shear in a basic flow that small perturbations will develop, through various instabilities, and eventually degenerate into turbulence. Some of these instabilities, at least during the initial stage of their development, may be understood within the framework of linear-instability theory, the main results of which will be recalled in this chapter. The nonlinear instability studies may prove to be useful in the future in understanding transition to turbulence, but, to date, they are still in progress and have not led to a unified theory of transition. An extremely useful tool to understand the transition, and assess the various theories proposed to describe it, is the direct-numerical simulations of the Navier-Stokes equations: numerous examples of these calculations will be given here.

Among the shear-basic flows which will be considered, we will make a distinction between the free-shear flows, such as mixing layers, jets or wakes on the one hand, and wall-bounded flows such as boundary layers, pipe flows or channel flows on the other hand. In free-shear flows, primary instabilities leading to the formation of coherent vortices are inviscid, in the sense that they are not affected by molecular viscosity, if it is small enough. In wall-bounded flows, on the contrary, the linear instabilities depend critically upon the viscosity (viscous instabilities), and vanish in the Euler case. Inviscid instabilities are much more vigorous than viscous ones. The present chapter will consider transition to turbulence both from the point of view of linear-instability theory of parallel flows, and experimental or numerical data. We will look also at transition in rotating shear layers and thermal convection.

It has to be emphasized that the concept of transition to turbulence is not very well defined: generally, what experimentalists call transition corresponds to the development of small-scale three-dimensional turbulence within the fluid. Actually, we will see that the whole pro-
cess of transition to turbulence may involve several successive stages: in the plane mixing layer behind a splitter plate for instance, these stages observed experimentally are: a) the growth of two-dimensional coherent vortices; b) the merging of these vortices together (pairing); c) a catastrophic breakdown into three-dimensional turbulence. But, as already discussed in Chapter I and further justified later on in the book, the two-dimensional coherent vortices, which appear before the transition to three-dimensional turbulence, may have some features of two-dimensional turbulence.

1 - The Reynolds number

The most famous (but certainly not the most characteristic) experiment on the transition to turbulence is the Reynolds experiment of a flow in a circular pipe (circular Poiseuille flow). Let $U$ be an average velocity of the flow across the tube section, $D$ the diameter of the tube, and $\nu$ the molecular viscosity. Reynolds (1883) introduced the non-dimensional parameter

$$ R = \frac{UD}{\nu} $$

and showed experimentally that there was a critical value of $R$ above which the flow inside the tube became turbulent. This was done by varying independently the velocity $U$, the diameter $D$ of the pipe, or considering fluids of various viscosities. The critical value $R_c$ found by Reynolds was of the order of 2000. For $R < R_c$, the flow remained regular ("laminar"), and for $R > R_c$ it became turbulent. These observations are corroborated by the measurements of the mean\(^1\) skin-friction coefficient at the wall $2\mu(d\bar{u}/dy)/\rho U^2$. Below $R_c$, it decreases as $R^{-1}$, as predicted theoretically with the laminar Poiseuille parabolic profile. At $R_c$ it undergoes an abrupt jump. Above $R_c$ its decrease is much more gentle than in the laminar case, and follows first a $R^{-1/4}$ law, as shown experimentally by Blasius (1913). Though, as we shall see, the notion of a critical Reynolds number is extremely ambiguous, this parameter can nevertheless be shown to characterize the relative importance of nonlinear interactions developing in the fluid: indeed, let us consider a fluid particle of transverse velocity $v$ in a the pipe. The time which is necessary for crossing is the "inertial" time

$$ T_{in} = \frac{D}{v} $$

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\(^{1}\) in a sense defined later