CHAPTER 2

GENERAL CURVED COMPOSITES

In this chapter we discuss a continuum theory for composite materials with general periodic or locally curved structures; this generalises the analysis given in the previous chapter. Again we use tensor notation and sum repeated indices over their ranges, but do not sum underlined repeated indices.

2.1. Some Preliminary Remarks on Geometry

In the previous chapter we considered periodic plane curved structures. However, the curving in the structure of unidirectional fibrous and layered composite materials can be in three dimensional space, not just periodic in a plane, – we will call this spatially periodic -, or it can be generally (i.e. spatially) local, or local in a plane as shown in Figs. 2.1.1, 2.1.2 and 2.1.3, respectively, at $x_1 = \text{const}$ and $x_3 = \text{const}$. In these figures, it is assumed that the reinforcing layers lie in planes parallel to the plane $Ox_1x_3$ and these materials consist of alternating packets which contain $N$ different layers, the thickness of which is denoted by $\Delta H$ given by formula (1.3.1).

![Fig.2.1.1. Part of a composite material with spatially periodic curved structure.](image1)

![Fig.2.1.2. Part of a composite material with spatially local curved structure.](image2)

2.2. Constitutive Relations

As in the previous chapter, we will use the Lagrangian coordinates $x_j$, which in their natural state coincide with Cartesian coordinates. Moreover, we assume that the equations (1.2.1)-(1.2.7) remain valid. Now we consider the formulation of the
elasticity relations for a composite material with arbitrarily curved structure. To do this, we isolate a representative curved packet of the composite material, shown in Figs. 2.2.1. and 2.2.2. In these figures the following notation is introduced: \( H \) is the characteristic vertical rise of the structural curve; \( \Lambda_1 \) and \( \Lambda_3 \) are the half-wavelengths of periodic curves in the directions of the \( O\overline{x}_1 \) and \( O\overline{x}_3 \) axes, respectively; \( \Delta H \) is the thickness of the representative packet; \( h' \) is the thickness determined by the expression (1.3.9). Note that for local curving in the structure of the composite, the meaning of the parameters \( \Lambda_1 \) and \( \Lambda_3 \) will be different.

![Fig.2.1.3. Part of a composite material with plane-local curved structure.](image)

We introduce a local coordinate system \( O'\overline{x}_1' x'_2 x'_3 \) with the origin on the median surface of the isolated representative packet; the axis \( O'\overline{x}_2' \) is directed along the normal vector \( \overline{r}_2' \), and the axes \( O'\overline{x}_1' \) and \( O'\overline{x}_3' \) along the tangential vectors \( \overline{r}_1' \) and \( \overline{r}_3' \) to this surface (Figs. 2.2.1 and 2.2.2). Note that, in the absence of curving, the directions of the vectors \( \overline{r}_1' \), \( \overline{r}_2' \), and \( \overline{r}_3' \) coincide with the directions of the vectors \( \overline{r}_1 \), \( \overline{r}_2 \), and \( \overline{r}_3 \), respectively (Figs. 2.2.1 and 2.2.2). The vectors \( \overline{r}_1' \), \( \overline{r}_2' \), and \( \overline{r}_3' \) are determined below.

The equation of the median surface of the selected representative packet we take as follows:

\[
x_2 = F(x_1, x_3) = \varepsilon f(x_1, x_3).
\]  

(2.2.1)

![Fig.2.2.1. The geometry of the representative packet for composite material with spatially periodic curved structure.](image)

![Fig.2.2.2. The geometry of the representative packet for composite material with spatially local curved structure.](image)