Although the laws of Newtonian mechanics are now acknowledged to be only approximately correct, classical mechanics is still the tool used by all save the nuclear physicists and the astronomers. No apology should be needed, therefore, for an investigation of the axioms of Newtonian mechanics. The fact that these axioms are not in exact accord with physical experience does not in any way imply that they cannot be stated clearly, precisely, operationally, and without internal contradiction. It is the aim of this paper to develop an axiom system that meets these standards.

There was widespread interest among physicists of the last half of the nineteenth century in the axiomatization of mechanics. Analysis of Newton's formulation in the *Principia* led to two lines of activity: first, the clarification of the concepts of 'mass' and 'force', and second, the elucidation of 'absolute' and 'relative' time, space, and motion. The latter line of investigation attracted the most interest, and was involved in the discovery of the special theory of relativity. This discovery, in turn, diverted attention almost completely from the axiomatization of classical mechanics. It is only very recently that any new attempts appear to have been made to further examine Newtonian mass and force.\(^1\)

Perhaps the most important product of the nineteenth century's studies in axiomatics, so far as mass and force are concerned, is Ernst Mach's reformulation of the laws of motion.\(^2\) The most careful statements of the axioms to be found in modern textbooks follow closely those of Mach. Hence, the present paper will begin with a re-examination of Mach's formulation and then proceed to a modified statement which, it is hoped, amends Mach at those points where he falls short of modern standards of 'operationalism'.

1. MACH'S STATEMENT OF THE LAWS OF MOTION

As already suggested, the laws of motion can be treated in two parts. First, rules can be stated for motion relative to a given reference system. Second,
rules can be stated for the selection of an appropriate reference system. (It will appear presently that these two parts are not entirely independent.)

With respect to the first part, Mach's principal contribution was to transform Newton's laws into kinematic definitions of force and mass. The first and second laws he treated not as physical hypotheses, but as definitions of force; and the third law follows as a consequence of his definitions of mass and force. All that remained of physical hypothesis as distinguished from definition in his version of Newton's system were the assumptions that:

(a) the mass ratios of two bodies A and B (say \( m_{A/B} \)) remain constant over time, and

(b) the mass ratios of bodies are transitive, i.e. \( m_{A/C} = m_{A/B} \cdot m_{B/C} \).

For purposes of reference we give here Mach's definitions and hypotheses as stated in 'The Science of Mechanics':

a. Experimental Proposition. Bodies set opposite each other induce in each other, under certain circumstances to be specified by experimental physics, contrary accelerations in the direction of their line of junction. (The principle of inertia is included in this.)

b. Definition. The mass-ratio of any two bodies is the negative inverse ratio of the mutually induced accelerations of those bodies.

c. Experimental Proposition. The mass-ratios of bodies are independent of the character of the physical states (of the bodies) that condition the mutual accelerations produced, be those states electrical, magnetic, or what not; and they remain, moreover, the same, whether they are mediately or immediately arrived at.

d. Experimental Proposition. The accelerations which any number of bodies A, B, C, ... induce in a body K, are independent of each other. (The principle of the parallelogram of forces follows immediately from this.)

e. Definition. Moving force is the product of the mass-value of a body into the acceleration induced in that body.

This statement certainly represents a substantial advance in clarity over Newton's formulation. One point deserves special notice: the definition of mass is purely inertial, that is, it makes no reference to the gravitational or other special forces acting on the system. In this respect its viewpoint differs rather fundamentally from that of general relativity theory.

In spite of their virtues, the laws stated by Mach raise some disturbing questions. First, in his definition of mass, Mach speaks of one body 'inducing' an acceleration in another. It is not clear what 'inducing' means. Two bodies are placed close to one another. Perhaps they are magnetized, or electrified, or even connected by a spring. It is observed that these bodies undergo certain accelerations (relative to some reference system). Are these the 'mutually induced accelerations' of which Mach speaks? How do we know that the accelerations produced are not due to the presence of other bodies in