2 Principles and practice of scaling laws

2.1 Introduction

This section covers the fundamentals of similarity, scaling, and the problems involved when models are used to predict full size machine performance. The fundamental hydrodynamic principles leading to Euler’s equation are introduced and applied to incompressible and compressible machines, and the problems associated with their use are discussed.

Three machine flow paths are possible – radial, mixed and axial flow, as indicated in Fig. 1.1 – and the flow direction and rotation is related to the machine action, either as energy input or as extraction.

Figure 2.1 Machine ‘black box’.
2.2 Performance laws

In a simple approach, a turbomachine can be considered as a black box. Fig. 2.1 describes a functional unit. A shaft transmitting torque is the external work output/input $P$, flow is passing through at a volume rate $Q$, the fluid is experiencing an energy change $gH$, and the rotor speed is $\omega$. How the energy $gH$ is achieved, and its relation to $Q$ and $P$, were discussed in Chapter 1. The quantities listed are all measurable externally, and can be used to indicate machine performance in a way relevant for the user.

If a pumping machine is considered, the input power $P$ is a function of flow rate $Q$, specific energy rise $gH$, machine size (characteristic dimension $D$), rotational speed $\omega$, and liquid properties $\mu$, $\rho$ and $\kappa$ (modulus). Using the principles of dimensional analysis, sets of non-dimensional groups may be obtained, one such being

$$\frac{P}{\rho \omega^3 D^5} = f \left[ \frac{Q}{\omega D^3}, \frac{gH}{\omega^2 D^2}, \frac{\rho \omega D^2}{\mu}, \frac{\rho \omega^2 D^2}{\kappa} \right]$$

The following may be noted for Equation 2.1:

(a) Grouping 1 is a power coefficient.
(b) Grouping 2 is often known as a flow coefficient $\phi$ (or, since $Q \propto VD^2$, and $\omega D \propto u$, $\phi = V/u$, which is a velocity coefficient).
(c) Since $\omega^2 D^2 \propto u^2$, grouping 3 can be written as $gH/u^2$, and without $g$ is known as $\psi$, the head coefficient.

![Figure 2.2 Constant speed characteristics of a pump.](image)