LIMITATIONS OF DETERMINISM IN CLASSICAL MECHANICS

The search for a sure explanation and prediction of phenomena is the main aim of science. Such an explanation can be obtained on the basis of deterministic laws and deductive reasoning only. On the other hand it is well known that in many fields of science deterministic description is impossible, so one has to use statistical laws which instead of exact predictions give probabilities of individual results. Therefore the application of statistical laws in science has stimulated many discussions, and numerical attempts at epistemological justification of them have been success less.

Before the rise of quantum mechanics it was widely believed that fundamental physical processes obeyed deterministic laws and physics was an ideal of scientific explanation. However, quantum theory is irreducibly statistical and that fact is still a serious problem for the philosophy of science. The sixth chapter of of *Science and Skepticism* by J. W. N. Watkins begins with a paragraph entitled “The Challenge of Microphysics”. The author writes:

I hold that here is only one serious challenge to the deductivist ideal for scientific explanation, but that it is very serious. Many of the theories in contemporary physics that conform admirably with the of (B') for depth and predictive power have an essentially statistical or probabilistic character; and it is widely held that such theories, although they explain much, cannot provide deductive explanation for what they explain.¹

Watkins assumes, in accordance with prevailing opinion that classical mechanics is determinist.

My aim is to show that, according to recent results of the abstract theory of dynamical systems, the role of determinism in classical mechanics is smaller than philosophers believe. The problem of statistical and deterministic laws and their interrelation appeared in physics in the second half of the nineteenth century, in connection with the rise and development of statistical mechanics. In that theory, fundamental interactions of a system of elements are mechanical, so they are determined and reversible, whereas the whole system develops in statistical and irreversible way. The problem was how to explain the origin and the nature of irreversibility and statistics in such a system. Many physicists and mathematicians have worked on that problem but during a hundred

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years they have failed to achieve its complete explanation. At the beginning it was believed that the deterministic theory (classical mechanics) described the nature of phenomena, while the statistical description was the result of scientists' ignorance. The statistics appeared when some degrees of freedom of the systems were neglected, when physicists passed from detailed microscopic to macroscopic description, when the procedure of coarse graining was used, etc. Summing up we can say, that in the course of one century determinism was treated as essential and statistics as superficial characterization of physical phenomena. Many paradoxes which appeared in those times were not fully solved.

The situation changed in the last thirty years, although we owe the first interesting results to Poincaré, who initiated the study of the mathematical stability of classical dynamical systems. He observed that the well-known school examples of mechanical processes, such as planetary motion, collision of balls, the harmonic oscillator etc., describe motions which are exceptionally stable. It means that if in the starting point two trajectories differ slightly, their difference remains small in the future. In other words, small errors in the measurement of the system's position and velocity lead to small changes in its future trajectory. That fact is of fundamental importance in practical applications of science (e.g. astronomy). It is called stability of motion. The stability depends on the kind of differential equation and a potential, both describing the class of processes. In classical mechanics the equation is universal, so the stability depends on the potential only.

Poincaré examined the problem in a general way. He defined an abstract space of dynamical systems and a measure on that space. He also proved that stable systems form in the general space a 0-measure set, i.e. almost all systems are unstable. It was a very important result giving physics new perspectives. The theory of dynamical systems was continued by Birkhof, Hopf and many others, who considered the first degree of instability — ergodicity. Ergodicity means that time averages and phase averages are equal in a system. Detailed studies of ergodicity lasted for decades. The main problem was the classification of ergodic systems. In 1958 Kolmogorov defined entropy as an invariant of automorphisms.

Kolmogorov's paper was the starting point of a new exciting period of rapid development of the theory. Mathematicians have defined several levels of system instability: ergodicity, mixing, K-flows, Anosov systems. Ornstein has proved that two Bernouli shifts with equal entropy are isomorphic. The mathematical theory of unstable dynamical systems uses many advanced methods and concepts, such as the spectral theory of operators, the measure theory,