USE OF A FOURIER DECOMPOSITION TECHNIQUE IN AQUATIC ECOSYSTEMS MODELLING

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Summary

A quasilinear system of ordinary first-order differential equations of the type frequently used in ecosystems modelling (including mathematical models of aquatic ecosystems) is considered. It is assumed that the system is subject to periodical changes in coefficients and/or right-hand sides (due to diurnal or seasonal character of the described ecological processes). The periodic component of the state variables caused by these disturbances is considered to be small enough to allow usage of first-order Taylor formulae. Under these assumptions a decomposition of the system dynamics into "the slow motion" component and first-order Fourier harmonics is performed. The resulting set of equations can be solved with large time steps, still preserving information on the periodic as well as the smooth average components of dynamical behavoir of the initial system. The performance of the method is evaluated using an algae growth equation, the only growth limiting factor being that of light availability. The results acquired suggest the proposed method is useful both for adjusting the average motion component and for evaluation of the diurnal dynamics of algae. Further uses of the method are discussed and proposed.

Keywords Numerical methods, nonlinear systems, mathematical modelling, aquatic ecosystems

1. Introduction

Mathematical modelling of aquatic systems (especially nutrient-phytoplankton models) is widely used in water resources planning, eutrophication analysis and policy making (Somlyódy and Van Straten, 1986). Recently, numerous advanced methodologies have become available in the field, allowing uncertainty analysis with Monte-Carlo simulations, dynamic programming and other techniques (M.B.Beck, 1987). Application of such methods requires repeated simulations of the system in question. Calibration of model parameters on the basis of measured data is likewise based on repeated simulations of the aquatic system. If diurnal changes of the component values are considerable, then the time step used in simulations must be small enough to reproduce within-day behavoir of state variables (for example, to account for the diurnal changes in solar radiation, which is important for phytoplankton growth). In pursuing increased time steps for simulation, the generally used approach is to integrate the right-hand sides of the governing equations over a daily time interval and then solve for the seasonal dynamic (Thomann and Mueller, 1987). However, the governing equations of the phytoplankton-nutrient system are nonlinear, and simple averaging of the right-hand sides can lead to poor results. But the

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diurnal changes of system state variables values are often small with respect to the daily averages, and this can be used in formulating equations for Fourier components of the system state variables. Exact reproduction of system dynamics can be achieved only if the series of Fourier components are infinite. In practice, however, it is often enough to retain zero and first order terms only. Closed systems of equations for Fourier components can then be constructed, using the assumption that diurnal variations of state variables are small relative to their daily average values. Zero-order components could then be used in the analysis as the daily averaged values of the corresponding state variables, and first-order components give an apprehension of the periodic dynamics of the system, including its magnitude and phase. This approach is similar to a well-known method from analytical mechanics dealing with a system disturbed by a periodical external force. Originally proposed by Bogolyubov and Krylov (1937), this method was later widely used in particle and plasma physics and in many other areas, but has not yet received yet appropriate attention in environmental disciplines. This approach of decomposition of system dynamics to a "smooth" averaged motion and first-order Fourier harmonics we shall call subsequently first-order Fourier decomposition. It can be used while modelling any processes described by a quasilinear system of differential equations, if this system is subject to periodic disturbances (for example, due to the diurnal character of the processes modelled).

An idea of fourier decomposition

Let us consider an ordinary first-order differential quasilinear equation of the form

$$\frac{dA(t)}{dt} = K(A,t)A(t) + b(t), \tag{1}$$

where $A(t)$ is a dependent variable, and t is the independent variable representing time. Assume that $K(A,t)$ and $b(t)$ are subject to periodic changes in time (with unit period for simplicity). We are aware that the solution of (1) bears a definite periodic character. Therefore the following substitution can be made:

$$A(t) = A_0(t) + A_1(t)\sin(2\pi t) + A_2(t)\cos(2\pi t), \tag{2}$$

where $A_0(t)$ is a "slow motion" component or "averaged" dynamics; $A_1(t)\sin(2\pi t)$ and $A_2(t)\cos(2\pi t)$ are first-order Fourier harmonics of periodic movement.

Naturally we should assume that the components $A_0(t)$, $A_1(t)$ and $A_2(t)$ do not change significantly during one period of the process cycle; furthemore, we assume that $A_1(t)$, $A_2(t) << A_0(t)$, so we could disregard higher order terms of the decomposition in subsequent derivations. The term dA_0/dt we will treat as having the same order as $A_1(t)$, $A_2(t)$.

For a derivation of a set of decomposed equations, first let us substitute (2) into (1). The result is

$$\frac{dA_0}{dt} + 2\pi A_1\cos(2\pi t) - 2\pi A_2\sin(2\pi t) + \ldots = KA(t) + b, \tag{3}$$

where ... denotes terms of higher order than zero and first with respect to $A(t)$ and which will be disregarded in the following considerations. Secondly, let us integrate (3) three