ABSTRACT. The recent development of a continuum model of the optical vibrations in quantum wells and superlattices which successfully reproduces the results of microscopic theory allows more reliable estimates of intra- and inter-subband scattering rates to be made. In this model the electron interacts in polar material with triple hybrids consisting of mixed LO, TO and IP (interface polariton) modes whose vibrational patterns are described by simple analytic expressions. The effect of hybridization depends upon the degree of polarity of the material and on conditions at the interfaces. For non-polar material there are only double hybrids consisting of LO and TO modes. For highly polar material (e.g. AlAs) and for polar slabs with free surfaces the interface modes are virtually decoupled from the confined LO/TO components. In polar superlattices with strongly hybridized modes there is strong anisotropy. All of this makes the scattering rates dependent on detailed structure and this opens up the possibility of phonon engineering. Some examples of scattering rates will be presented.

1. Introduction

Quantum-well devices typically involve hot electrons, whose energy relaxation takes place principally by the net emission of optical phonons. The rate at which this occurs affects the speed of the device, and it is extremely useful to have available a reliable theory of the electron-phonon interaction which does not involve excessive computational time in order to model the performance of the device. While electron states have been reasonably well described by envelope-function formalism, phonon states have required extensive numerical analysis of the lattice dynamics of the heterostructure in order to describe the effects of confinement. Recently, a number of analytical models of phonon confinement have been advanced [1-3] which to varying degree reflect the mixing which occurs between longitudinally-polarized optical (LO) and interface polariton (IP) modes as revealed by microscopic calculations [eg. 4-6]. These models ignored the role of transversely-polarized optical (TO) modes, which were shown to be essential ingredients in satisfying mechanical boundary conditions in non-polar material [7] and in satisfying both mechanical and electrical boundary conditions in polar material [8,9,10]. A continuum model of confined phonons now exists which describes confinement in terms of triple hybrids (LO/TO/IP) in the case of polar material, and double hybrids (LO/TO) in the case of non-polar material, and it allows a straightforward analytic formulation of the interaction with electrons to be made.
A brief description of this model is given in Section 2 and some of the features of the unscreened interaction of hybrid phonons with electrons are outlined in Section 3.

2. Optical-Mode Hybrids

The equation of motion which describes long-wavelength optical vibrations in bulk, isotropic material is

\[
\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = -\rho \omega_{\text{TO}}^2 \mathbf{u} + e_i \mathbf{E} - c_{11} \nabla (\nabla \cdot \mathbf{u}) + c_{44} \nabla \times \nabla \times \mathbf{u}
\]  

where \( \mathbf{u} \) is the relative displacement of the ions, \( \rho \) is the reduced density, \( \omega_{\text{TO}} \) is the natural frequency, \( e_i \) is the effective ionic charge density, \( \mathbf{E} \) is the macroscopic electric field, and \( c_{11}, c_{44} \) are optical-mode elastic constants which quantify the macroscopic stress in the medium and determine the dispersion. Elastic anisotropy can easily be taken into account, but here we will ignore it from the outset since, ultimately, averages over direction would have to be taken to describe scattering rates. It is assumed that a continuum picture suitable for bulk material is applicable to the quantum-well, superlattice and slab cases. Coupled with the equation of motion are Maxwell's equations.

The displacements can be factorized into longitudinal (\( \nabla \times \mathbf{u}_L = 0 \)) and transverse (\( \nabla \cdot \mathbf{u}_T = 0 \)) parts, and three distinct travelling-wave solutions emerge, one longitudinal (LO) and two transverse (TO and IP). With reference to an interface these solutions further divide into two types: those with polarizations lying in the plane of incidence (p-modes) and those with polarization at right-angles (s-modes). The latter can only be a TO mode, and it has no difficulty in satisfying boundary conditions without mixing. The same is not true for p-modes. The requirement that both mechanical and electromagnetic boundary conditions be satisfied forces these modes to form a linear combination, leading to a triple hybrid. The principal properties of these modes are given in Table 1.

**TABLE 1. Principal properties of optical modes**

<table>
<thead>
<tr>
<th>Mode</th>
<th>Dispersion</th>
<th>Electric Field</th>
<th>Permittivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>( \omega^2 = \omega_{\text{LO}}^2 - \nu_L^2 k^2 )</td>
<td>( E_L = -\frac{e_i}{\varepsilon_\infty} \mathbf{u}_L )</td>
<td>( \varepsilon(\omega) = 0 )</td>
</tr>
<tr>
<td>TO</td>
<td>( \omega^2 = \omega_{\text{TO}}^2 - \nu_T^2 k^2 )</td>
<td>( E_T = 0 )</td>
<td>( \varepsilon(\omega) \to \infty )</td>
</tr>
<tr>
<td>IP</td>
<td>( \omega^2 = k^2/(\mu_0 \varepsilon(\omega)) )</td>
<td>( E_p = s \mathbf{E}_L )</td>
<td>( \varepsilon(\omega) = \varepsilon_\infty \frac{\omega_{\text{LO}}^2 - \omega^2}{\omega_{\text{TO}}^2 - \omega^2} )</td>
</tr>
</tbody>
</table>

\( \omega_{\text{LO}}^2 = (\varepsilon_\infty/\varepsilon_\infty) \omega_{\text{TO}}^2 , \quad e_i^2 = \varepsilon_\infty^2 \rho (\omega_{\text{LO}}^2 - \omega_{\text{TO}}^2) , \quad s = (\omega^2 - \omega_{\text{TO}}^2)/(\omega_{\text{LO}}^2 - \omega_{\text{TO}}^2) \),

\( \nu_L^2 = c_{11}/\rho , \quad \nu_T^2 = c_{44}/\rho , \quad \varepsilon_s , \quad \varepsilon_\infty = \text{low and high frequency permittivities} \),

\( \mu_0 = \text{permeability} \).