Shear deformation and sandwich configuration

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5.1 INTRODUCTION

Classical plate theory (CPT) is generally credited to Kirchhoff over 125 years ago. It is based on the following simplifying hypotheses due to Kirchhoff:

H1. plane sections prior to deformation remain plane and normal to the deflected reference surface; and
H2. the thickness does not change during deformation.

The corollaries of these respective hypotheses are:

C1. there is no transverse (or thickness direction) shear strain; and
C2. there is no thickness normal strain.

The Kirchhoff hypotheses have been adequate for most structural applications of plates constructed of homogeneous isotropic materials. (The few exceptions are for the case of very thick plates.) In contrast, owing to the low shear modulus of most matrix materials relative to the elastic modulus of the fibres, typical fibre-reinforced composite materials have macroscopic transverse shear moduli that are low relative to the in-plane elastic moduli. This means that transverse shear deformation effects are usually more significant in laminated composite-material plates than in homogeneous isotropic plates of the same relative thickness.

Sandwich plates consist of one or more thick, flexible cores to which are bonded thin relatively stiff facings. The facings provide the bending stiffness (like the flanges of an I beam), while the core serves to transfer the loading from facing to facing (like the web of an I beam). Owing to the nature of the typical core materials used, thickness shear deformation is important in connection with stiffness, while both thickness shear and thickness normal deformation play very important roles in the kind of local buckling known as face wrinkling.
The scope of the present chapter includes buckling and postbuckling of both compact laminated and sandwich plates subjected to static mechanical loads, temperature and/or moisture changes, and dynamic loadings. Emphasis is placed upon the transverse shear effects in laminated plates and both transverse shear and transverse normal effects in sandwich plates.

The coordinate system used is the conventional right-handed one for plate theory. The reference plane is taken to be the midplane and \( x \) and \( y \) are the Cartesian coordinates in that plane. The coordinate \( z \) is measured downward normal to the midplane. Particular attention is directed to rectangular-planform plates and dimensions \( a \) and \( b \) refer respectively to the length (in the \( x \) direction) and width (in the \( y \) direction). Unless specified otherwise, it will be assumed that \( ab \geq 1 \).

The material of each layer or ply is assumed to be orthotropic with respect to its material-symmetry axes. These axes are denoted by \( L \) (longitudinal – in the fibre direction), \( T \) (transverse in the plane), and \( N \) (normal to the plane of the layer). Thus, the in-plane elastic moduli are \( E_L \) and \( E_T \), the in-plane shear modulus \( G_{LT} \), the thickness shear moduli \( G_{LN} \) and \( G_{TN} \), and the thickness normal modulus \( E_N \). The in-plane Poisson’s ratios are denoted by \( \nu_{LT} \) and \( \nu_{TL} \), and the out-of-plane Poisson’s ratios by \( \nu_{LN}, \nu_{NL}, \nu_{TN}, \) and \( \nu_{NT} \). Assuming that the elastic properties are the same in tension and compression, symmetry of the stiffness array requires that an orthotropic material can have only nine independent elastic constants. Thus, the twelve elastic constants mentioned must be interrelated by the following so-called reciprocal relations:

\[
\begin{align*}
\frac{\nu_{LT}}{E_L} &= \frac{\nu_{TL}}{E_T} \\
\frac{\nu_{LN}}{E_L} &= \frac{\nu_{NL}}{E_N} \\
\frac{\nu_{TN}}{E_T} &= \frac{\nu_{NT}}{E_N}
\end{align*}
\] (5.1)

If an orthotropic layer is oriented at an acute angle \( \theta \) from the in-plane reference axis \( (x) \) by rotation about the normal axis \( (z) \), the layer behaves as if it were monoclinic. Then its stress–strain relation can be expressed as

\[
\begin{pmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xz} \\
\tau_{yz}
\end{pmatrix} = 
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\
C_{21} & C_{22} & C_{23} & 0 & 0 & C_{26} \\
C_{31} & C_{32} & C_{33} & 0 & 0 & C_{36} \\
C_{44} & C_{45} & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\begin{pmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xz} \\
\gamma_{yz}
\end{pmatrix}
\] (5.2)