The Southern Oscillation (SO) is the most dominant mode in short-term climate variations over the globe, particularly in the tropics and midlatitudes, accounting for a significant portion of the variance in the global climate system. In a compact expression, the SO is a large-scale phenomenon in which atmospheric masses are exchanged between centres in the Pacific and Indian Oceans. Climate variability and hydrological cycle in many parts of the world (e.g. southern Europe) are intimately related to the SO. The state of the SO is generally described by an index (SOI), which is the normalized sea level pressure difference between Tahiti and Darwin, Australia.

Using a time-domain approach, autoregressive-moving average (ARMA) processes are applied to model and predict the SOI on a monthly and seasonal basis. Based on the sample autocorrelation function, sample partial autocorrelation function, and the Bayesian Information Criterion, an ARMA(1,7;1) process is found to fit the monthly SOI and an AR(3) process to fit the seasonal SOI adequately (Chu and Katz, 1985). Figure 1a shows the theoretical autocorrelation function (broken curve) for an ARMA (1,7;1) process and the sample autocorrelation function of the monthly SOI. Figure 1b shows the theoretical autocorrelation function (broken curve) for an AR(3) process and the sample autocorrelation function of the seasonal SOI.

Regarding prediction, Figure 2 displays the one-season-ahead forecast values of the seasonal SOI for the AR(3) model, along with the actual observations from Summer 1982 to Winter 1984. Over this period, unprecedented large anomalies occurred between Summer 1982 – Spring 1983. With the exception of Summer 1982, a close correspondence between the observed and predicted series is readily seen, both in the direction of variation and in magnitude.

Besides their use as diagnostic and forecasting tools, time series models have other potential uses. For instance, Chu and Katz (1987) developed a method to provide a measure of actual, rather than potential predictability of a climate variable on the basis of its past history alone. This method explicitly estimates how well such a time series model can forecast future behaviour. Specifically, the proportion of variance explained by the l-step ahead forecast is:

\[ \frac{\sum (\hat{y}_t - \bar{y})^2}{\sum (y_t - \bar{y})^2} \]

\[ \text{Equation 1.2} \]

\[ \text{Equation 1.3} \]
$$\lambda_{\ell} = 1 - \frac{\nu(\ell)}{\sigma^2} \quad \ell = 1, 2, \ldots$$

where $\nu(\ell)$ denotes the variance of the $\ell$-step ahead forecast error, and $\sigma^2$ the variance of the $X_t$ process as

$$X_t = \sum_{i=1}^{p} \phi_i X_{t-i} + a_t - \sum_{j=1}^{q} \theta_j a_{t-j}$$

Note that $0 \leq \lambda_{\ell} \leq 1$, with $\lambda_{\ell}=1$ for the case of perfect predictability and $\lambda_{\ell}=0$ for the case of no predictability.

Figure 1. (a) Sample autocorrelation function of the monthly SOI and the theoretical autocorrelation function (broken line) for the ARMA $(1,7;1)$ process. (b) Sample autocorrelation function of the seasonal SOI and the theoretical autocorrelation (broken line) for the AR(3) process.