Let \( x_1, x_2, \ldots \) be independent random variables with expectations

\[
\mathbb{E} x_n = 0, \quad \mathbb{E} x_n^2 = 2b_n, \quad \mathbb{E}|x_n|^3 = d_n;
\]

and suppose that the ratios \( d_n/b_n \) are smaller than a certain fixed constant

\[
d_n/b_n \leq \mu. \tag{1}
\]

Set

\[
S_n = x_1 + \ldots + x_n, \quad t_n = b_1 + \ldots + b_n.
\]

An important problem in probability theory is to study the dependence of the sums \( S_n \) as \( n \to \infty \), on the properties of \( t_n \) as \( \mu \to 0 \). On the basis of the now classical law of Lyapunov \(^1\) we have

\[
P\{a < S_n < b\} = \frac{1}{\sqrt{2\pi t_n}} \int_a^b e^{-s^2/4t_n} ds + \theta R(t_n, \mu), \quad |\theta| \leq 1, \tag{2}
\]

and \( R(t_n, \mu) \to 0 \) uniformly with respect to \( \mu \) if \( t_n \) is greater than some constant \( T \).

Thus, for fixed \( n \) we have a formula for the asymptotic behaviour of \( S_n \).

Consider the following problem. Let \( a(t) \) and \( b(t) \) be functions of the parameter \( t \). What is the probability that all the inequalities

\[
a(t_k) < S_k < b(t_k), \quad k = 1, 2, \ldots, n \tag{3}
\]

hold? Assume that \( a(t) \) and \( b(t) \) are continuously differentiable and that

\[
a(t) < b(t), \quad a(0) < 0 < b(0).
\]

Then we can obtain an asymptotic solution of our problem similar to (2).

All the sums \( S_1, S_2, \ldots, S_n \) (up to \( S_n \)) can be divided into the following three classes \( K, K_1 \) and \( K_2 \):

\( K \) contains those sums for which all the inequalities (3) hold;


$K_1$ contains those sums for which there exists a $k$ such that all inequalities
\[ a(t_i) < S_i < b(t_i), \quad i = 1, 2, \ldots, k - 1. \]
\[ S_k \leq a(t_k) \]
hold;

$K_2$ contains those sums for which there exists $k$ such that the inequalities
\[ a(t_i) < S_i < b(t_i), \quad i = 1, 2, \ldots, k - 1, \]
\[ b(t_k) \leq S_k \]
hold. Denote the probabilities corresponding to these sets by $P_n, P_n^{(1)}, P_n^{(2)}$.

Finally, denote by $P_n(x, y)$ the probability of the event
\[ u(t_k) < S_k < b(t_k), \quad k = 1, 2, \ldots, n - 1, \]
\[ x < S_n < y. \]

Clearly
\[ P_n = P_n \{ a(t_n), b(t_n) \}. \]

The inequalities
\[ t > 0, \quad a(t) < s < b(t) \]
single out a region $G$ in the $(s, t)$-plane. Denote by $g(s_0, t_0; s, t)$ the Green's function for the heat equation
\[ \frac{\partial f}{\partial t} = \partial^2 f / \partial s^2 \]
in $G$ and set
\[ g(s, t) = g(0, 0; s, t), \quad \partial g(s, t) / \partial s = u(s, t), \]
\[ v_1(t) = -u[a(t), t], \quad v_2(t) = u[b(t), t]. \]

**Theorem.** The following asymptotic formulas hold
\[ P_n(x, y) = \int_x^y g(s, t_n)ds + \Theta R(t_n, \mu), \quad (5) \]
\[ P_n^{(1)} = \int_0^{t_n} v_1(t)dt + \Theta_1 R_1(\mu), \quad (6) \]
\[ P_n^{(2)} = \int_0^{t_n} v_2(t)dt + \Theta_2 R_2(\mu), \quad (7) \]
\[ |\Theta| \leq 1, \quad |\Theta_1| \leq 1, \quad |\Theta_2| \leq 1, \]