Chapter 4:

Fuzzy Filter Functions and Convergence
§1. Introduction

In this chapter we describe a fuzzy filter functor in a general framework of set functors. The general theory includes generalized Cauchy spaces, together with a construction for completions, and generalized pseudo-topologies, which in the case of the fuzzy filter functor results in a development of fuzzy convergence structures.

The outline of the chapter is as follows. In section 2, we describe functor structures as defined through covariant set functors. Examples include powerset, ideal and filter functors, and also the fuzzy filter functor. Section 3 deals with functors from the category SET of sets to the category PROSET of preordered sets. Here we also present Urysohn modifications of functors, which overcomes a lattice-theoretical barrier in connection with the completion construction. Section 4 presents details about the fuzzy filter functor. In section 5, we develop fuzzy convergence, which also includes fuzzy topologies. Incidentally, concerning fuzzy topologies, the question about constants being open or not turns out to relate to the connectedness of the fuzzy filter functor. Section 6 contains the completion construction, and the chapter is concluded with section 7 on monads. The notion of a monad relates to the development of regularity and iteratedness, as well as to compactifications.

§2. Functor Structures

Many structures can be described by means of set functors, i.e. by functors the domain and codomain of which is the category SET of all sets. A necessary and sufficient condition for all structures of one and the same type to be describable by set functors has been given by [Kučera and Pultr 1972/1973] (see also [Adámek 1983]). To make this treatment simple we will here consider structures only on sets. However, we point out the fact that in the more general heterogeneous case, namely when the structures are e.g. on families of sets, a lot of interesting examples of structure types appear which are important in applications (see e.g. [Burmeister 1986, Gäbler and Gäbler 1989, Gäbler 1984]). In the heterogeneous