Three Dimensional Composite Elements for Finite Element Analysis of Anistropic Laminated Structures

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ABSTRACT

In this paper, a 3-D, 8-node and a 3-D, 20-node composite finite element are derived for finite element analysis of composite laminates based on a composite variational principle and a partial stress field. By taking three in-plane strains $\epsilon_x$, $\epsilon_y$, $\epsilon_{xy}$ and three transverse stresses $\sigma_z$, $\sigma_{yz}$, $\sigma_{xz}$ as the basic variables, the continuity of the transverse stresses $\sigma_z$, $\sigma_{yz}$, $\sigma_{xz}$ across the laminate thickness is assured a priori. A new procedure, called iso­function method, has been developed to form the partial stress field based on the assumed displacement field.

The finite element analysis of the interlaminar stresses in a three layer laminated beam subjected to bending loads is done with the composite finite elements. The results show the clear advantages of these three dimensional composite finite elements.

INTRODUCTION

The finite element analysis of the structures of composite materials has the main difficulty called continuous and discontinuous problem. Based on equilibrium and compatibility at the interlayer surfaces of the laminated structure, the three in-plane strains and three transverse stresses must be continuous. The other three in-plane stresses and three transverse strains may be described by a finite discontinuity which is caused by the abrupt
change of material property or orientation of different laminates[1].

The displacement formulated finite element method[8] has difficulties satisfying the above conditions. Since 1970's, many techniques have been proposed to deal with this problem. It can be seen from the works of previous researchers that by taking the six globally continuous components of stress and strain (three transverse stresses, three in-plane strains) as basic variables and by treating the interlaminar surfaces as boundaries with six continuity conditions between every two adjacent layers, the formulated variational principle automatically satisfies the three transverse stresses and three in-plane strains continuity conditions and permit possible discontinuities of three in-plane stresses and three transverse strains.

However, in order to derive finite elements based on the above mentioned variational principle, it is necessary to assume partial stress fields a priori. Finite element formulations involving an assumed stress field are cursed with zero-energy modes. These zero-energy modes have plagued the hybrid finite element method since the beginning. Previous researchers have proposed many techniques to overcome the zero-energy modes. Pian and Tong[3] proposed that the number m of the undetermined coefficients in the stress field should satisfy the relation $m \geq n - r$, where $n$ is the total degrees of freedom and $r$ is the degrees of rigid body mode. Atluri et.al[5, 6], using a symmetric group theory, identified the different possible modes that exist for a certain type of element. Recently, Huang [9] proposed a modal analysis technique to obtain the stress modes from the deformation modes of an assumed displacement field. Huang’s method has some degree of success for a certain type of finite element. It is also restricted to isotropic materials. The application to anisotropic materials is limited.

Thus, to develop a finite element method for stress analysis of composite laminates, a technique of taking three transverse stresses and three in-plane strains as basic variables has to be used; to form the new finite element based on the technique, a procedure to construct the partial stress field has to be developed to ensure no zero energy modes exist in the element.

THE ESTABLISHMENT OF COMPOSITE FINITE ELEMENT EQUATIONS

A composite laminate exhibits discontinuities. However, from consideration of equilibrium and comparibility, the three transverse stresses $\sigma_x$, $\sigma_y$, $\sigma_z$ and three in-plane strains $\epsilon_x$, $\epsilon_y$, $\epsilon_z$ must exhibit global continuity. As such neither displacement formulation method nor stress formulation method can satisfy the six continuity conditions a priori.